# DIGITAL COMPUTER PROGRAM FOR GENERAL MISSILE DYNAMICS SIMULATION

## (N-STAGE)

## SUMMARY OF EQUATIONS

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This report presents a summary of the mathematical equations contained in the N-STAGE missile simulation program. This program, written for the IBM 7090 digital computer, is designed to be a flexible, high speed computation tool, useful for guidance system analysis, trajectory design, hardware evaluation, or post-flight analysis of any current or proposed missile system. The simulation is generally complete and in most instances accuracy will be limited only by the accuracy of the input data.

Currently, work is proceeding which will add the effects of missile rotational inertia to the simulation.

The mathematical model used is fundamentally simple. The forces acting on an assumed "point mass" are computed and divided by the mass of the missile to obtain accelerations. These accelerations are added vectorially to that of gravity and numerically integrated to obtain a velocity increment over a short period of time. The velocity is numerically integrated to obtain a position increment for the same period of time. As this process is repeated, the normal output of the simulation is the time history of a trajectory.

Discontinuities in the trajectory, such as stagings, jettisons, etc., are functions of input data and are treated logically by the program.

In addition to the quantities necessary to the simulation, other descriptive quantities are computed and printed.

For design and post-flight analysis problems, automatic iterations (over trajectories or parts of trajectories) may be made by the program. The iteration techniques allow any number of input parameters to be found, with reference to values for any number of fundamental or descriptive quantities. Maximization or minimization may also be accomplished, with or without the presence of other constraints.

Although numerical integration is at the heart of the program, no statement of the integration techniques used is here made. The interested reader is referred to any standard numerical analysis text for a discussion of Runge-Kutta and Adams-Moulton integration techniques.

# Derivatives of Equations of Motion

All of the basic computations are carried out in a Cartesian inertial system having its origin at the center of the reference ellipsoid. The x and y axis pass through the equator and the z axis is along the polar axis. (Fig. 1)

Missile attitude determines the directions of the force vectors. Mutually perpendicular unit vectors are defined such that $\vec{\xi}$ points along the missile roll axis, $\vec{\eta}$ lies in the pitch plane, and $\vec{\zeta}$ lies in the yaw plane. (Fig. 1)

<u>Thrust and Mass Flow</u>

are given by

$$F_{\xi F} = K_F F_o$$

$$\dot{M} = K_{\dot{M}} \dot{M}_o$$

where

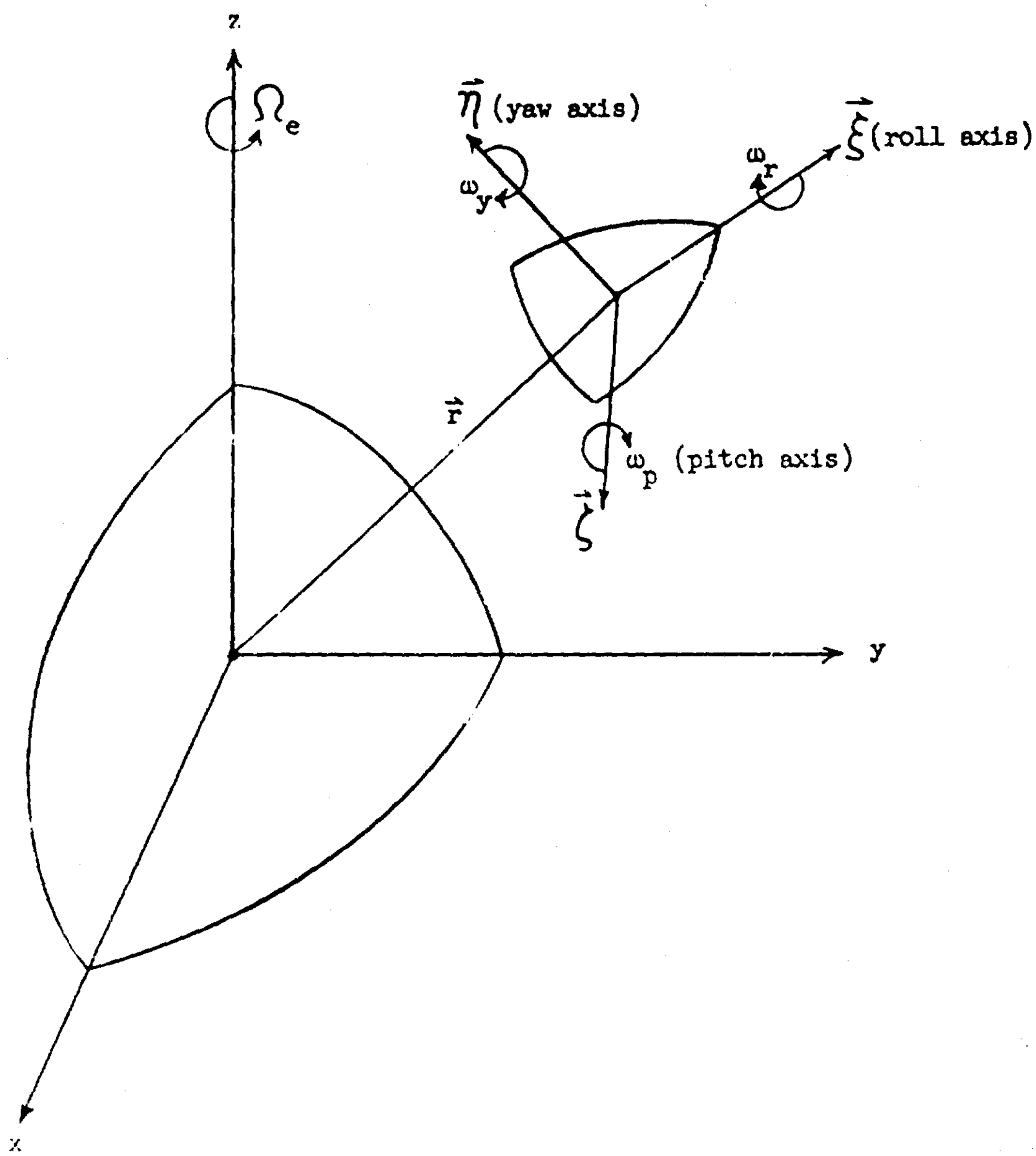
$F_o$, $\dot{M}_o$ may result from a table interrogation or from the Atlas Influence Coefficient Engine Model.

Velocity relative to air mass is

$$\vec{V}_a = \begin{bmatrix} \dot{x} + \Omega_e y - V_{wx} \\ \dot{y} - \Omega_e x - V_{wy} \\ \dot{z} + 0 - V_{wz} \end{bmatrix}$$

where wind velocity components are

$$V_{wx} \quad -W_\phi \frac{xz}{|\vec{r}| \left[x^2 + y^2\right]^{1/2}} - W_\theta \frac{y}{\left[x^2 + y^2\right]^{1/2}}$$




CARTESIAN INERTIAL COORDINATE SYSTEM WITH ATTITUDE UNIT VECTORS

FIGURE 1

## Derivatives of Equations of Motion

<u>Thrust and Mass Flow</u> (Cont'd)

$$V_{wy} = -W_{\phi} \frac{yz}{|\vec{r}| \left[x^2 + y^2\right]^{1/2}} + W_e \frac{x}{\left[x^2 + y^2\right]^{1/2}}$$

$$V_{wz} = W_{\phi} \frac{\left[x^2 + y^2\right]^{1/2}}{|\vec{r}|}$$

and

$$W_{\phi} = -V_w K_p \cos A_z \qquad \text{(north component of wind)}$$

$$W_e = -V_w K_p \sin A_z \qquad \text{(east component of wind)}$$

Atmospheric quantities $\rho$ (slugs/ft$^3$), C (ft/sec), T (deg Rankine) and P (lbs/in$^2$) are determined by approximation to the ARDC model of 1959 and are available to 2.3 million feet of altitude.

Radius at sea level

$$R_{SL} = \bar{A} \left[1 - \frac{1}{2} \left(k \frac{z}{|\vec{r}|}\right)^2 + \frac{3}{8} \left(k \frac{z}{|\vec{r}|}\right)^4\right]$$

where

$$k^2 = \frac{2e - e^2}{(1 - e)^2}$$

$e$ ellipticity (flattening)

Altitude

$$h = |\vec{r}| - R_{SL}$$

## Derivatives of Equations of Motion

Thrust and Mass Flow (Cont'd)

Mach number

$$N_m = \frac{|\vec{V}_a|}{C}$$

Dynamic pressure

$$Q = \frac{1}{2} \rho \left|\vec{V}_a\right|^2$$

Aerodynamic Drag

$$F_{\xi_D} = C_D \, Q \, S$$

where $C_D$ is obtained from a table, normally as a function of $N_m$.

Center of Gravity Offset Force

$\xi_{CG}$ (See Fig. 3) is obtained directly from a table or $\bar{\xi}_{CG}$ is obtained from a table and

$$\xi_{CG} = L - \bar{\xi}_{CG}$$

where $\bar{\xi}_{CG}$ is nose to center of gravity distance along $\vec{\xi}$ axis. The CG distances in the $\eta$ direction ($\eta_{CG}$) and $\zeta$ direction ($\zeta_{CG}$) are tabular.

Center of gravity offset forces due to drag in $\eta$ and $\zeta$ directions are

$$F_{\eta_D} = F_{\xi_D} \left(\frac{\eta_{CG}}{\xi_{CG}}\right)$$

$$F_{\zeta_D} = F_{\xi_D} \left(\frac{\zeta_{CG}}{\xi_{CG}}\right)$$

## Derivatives of Equations of Motion

### Center of Gravity Offset Force (Cont'd)

Center of gravity offset forces due to thrust misaligment in $\eta$ and $\zeta$ directions are

$$F_{\eta_F} = F_{\xi_F} \left(\frac{\eta_{CG}}{\xi_{CG}}\right)$$

$$F_{\zeta_F} = F_{\xi_F} \left(\frac{\zeta_{CG}}{\xi_{CG}}\right)$$

### Aerodynamic Normal Force

Pitch and yaw components of angle of attack ($\alpha$), angle between $\vec{\xi}$ and $\vec{V}_a$, are determined from

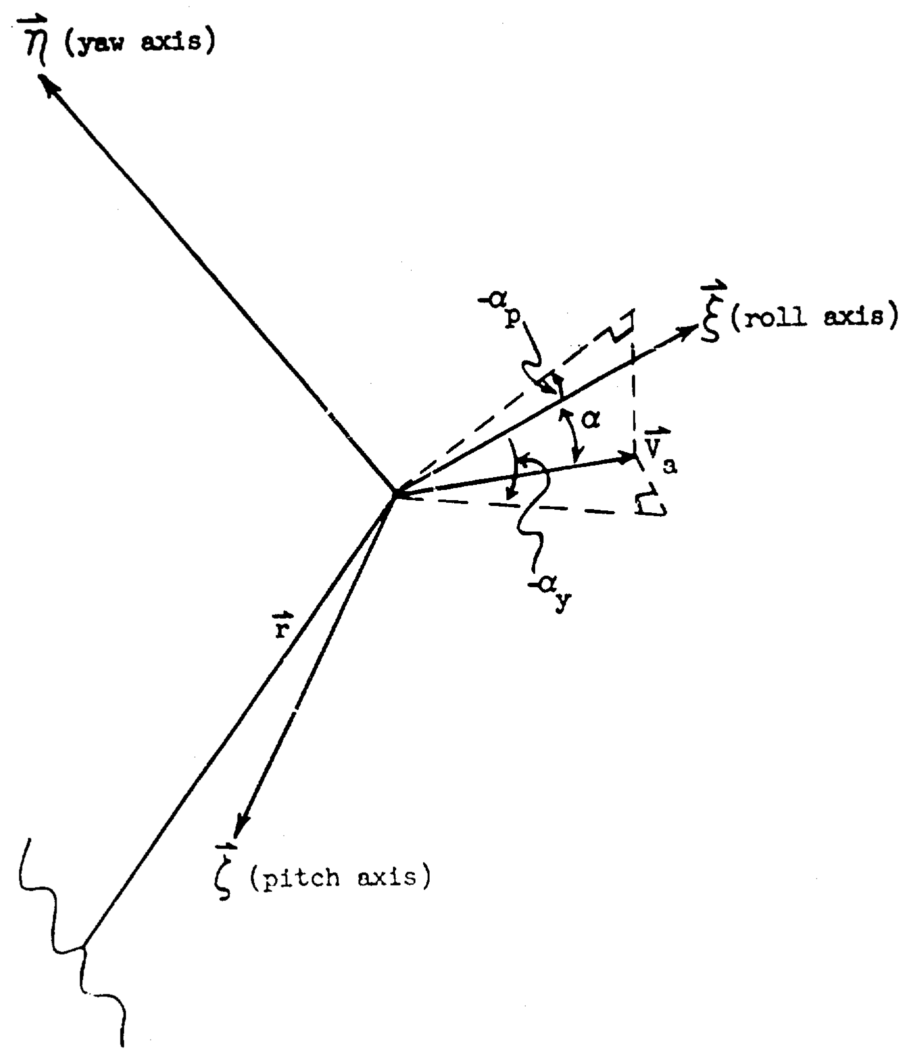
$$\tan \alpha_p = -\frac{\vec{\eta} \cdot \vec{V}_a}{\vec{\xi} \cdot \vec{V}_a} \doteq -\frac{\vec{\eta} \cdot \vec{V}_a}{|\vec{V}_a|}$$

(the approximate form is useful for small values of $\alpha$)

$$\tan \alpha_y = -\frac{\vec{\zeta} \cdot \vec{V}_a}{\vec{\xi} \cdot \vec{V}_a} \doteq -\frac{\vec{\zeta} \cdot \vec{V}_a}{|\vec{V}_a|}$$

$$\tan \alpha = \left[\tan^2 \alpha_p + \tan^2 \alpha_y\right]^{1/2}$$

If the approximation is used, $\alpha_p$ and $\alpha_y$ are assumed equal, respectively, to $\tan \alpha_p$ and $\tan \alpha_y$.




ANGLES OF ATTACK

FIGURE 2

## Derivatives of Equations of Motion

**Aerodynamic Normal Force** (Cont'd)

Aerodynamic normal force coefficients in the pitch and yaw planes ($C_{N_p}$, $C_{N_y}$) may be presented as tabular functions of $\alpha_p$, $\alpha_y$ and $N_m$. When $C_{N_{p_\alpha}}$ and $C_{N_{y_\alpha}}$ are presented

$$C_{N_p} = C_{N_{p_\alpha}} \alpha_p$$

$$C_{N_y} = C_{N_{y_\alpha}} \alpha_y$$

or, a tabular normal force coefficient may be entered directly or as a function of total angle of attack:

$$C_N = C_{N_\alpha} \alpha$$

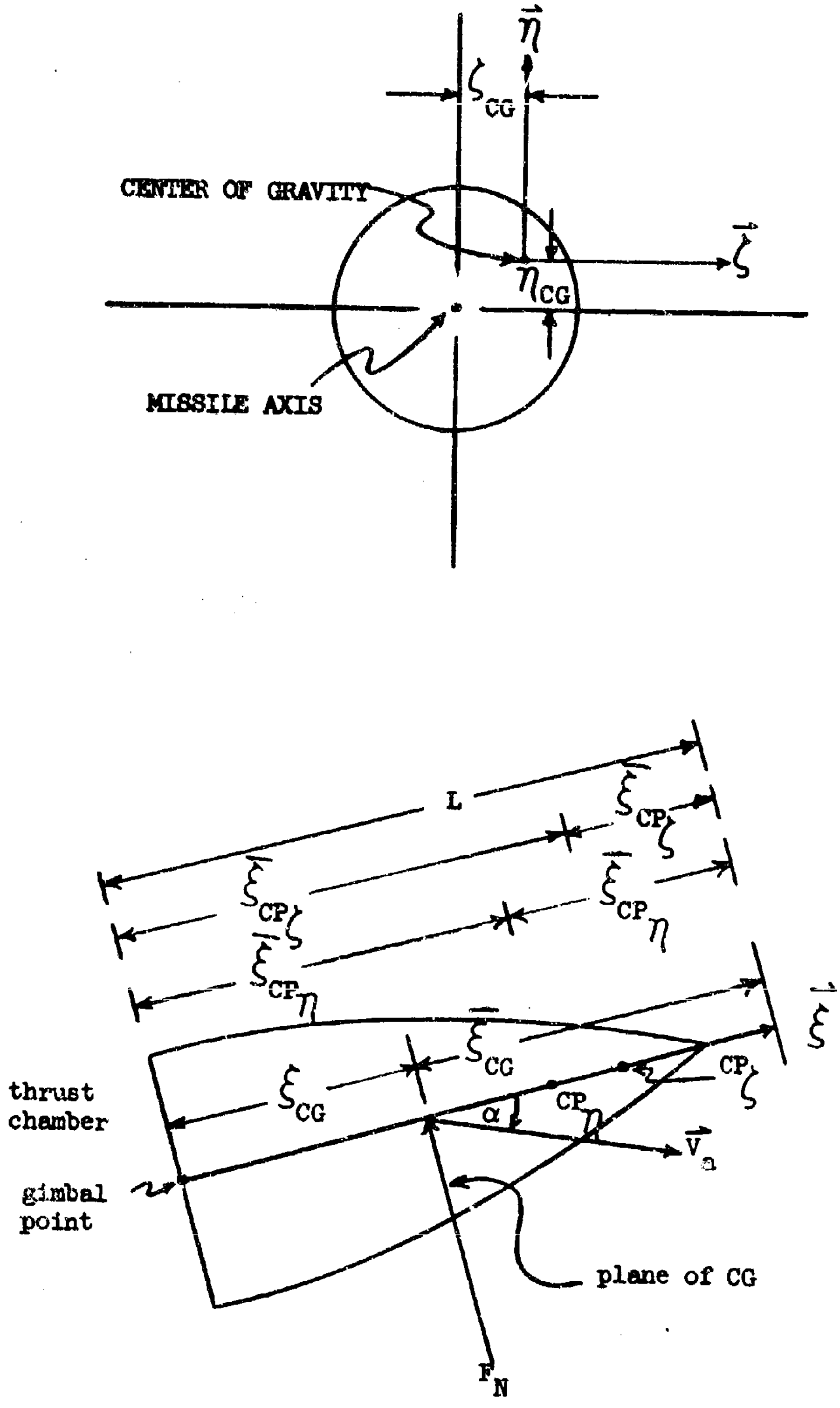
$$C_{N_p} = \frac{\tan \alpha_p}{\tan \alpha} (C_N)$$

$$C_{N_y} = \frac{\tan \alpha_y}{\tan \alpha} (C_N)$$

Either $\xi_{cp_{\eta,\zeta}}$ are obtained directly from a table or $\bar{\xi}_{cp_{\eta,\zeta}}$ are obtained from a table and

$$\xi_{cp_{\eta,\zeta}} = L - \bar{\xi}_{cp_{\eta,\zeta}}$$

where $\bar{\xi}_{cp_{\eta,\zeta}}$ are the nose to center of pressure distances in the pitch and yaw planes [illegible] $\vec{\xi}$ axis (Fig. 3)




CENTER OF GRAVITY

FIGURE 3

## Derivatives of Equations of Motion

### Aerodynamic Normal Force (Cont'd)

Aerodynamic normal forces in $\vec{\eta}$ and $\vec{\zeta}$ directions are

$$F_{\eta_N} = C_{N_p} \frac{\xi_{cp_\eta}}{\xi_{cg}} QS$$

$$F_{\zeta_N} = C_{N_y} \frac{\xi_{cp_\zeta}}{\xi_{cg}} QS$$

If

$\xi_{cp_\zeta}$ (or $\bar{\xi}_{cp_\zeta}$) is not supplied, $\xi_{cp_\eta}$ (or $\bar{\xi}_{cp_\eta}$) is used in both planes.

### Resultant Forces

$$F_\xi = F_{\xi_F} - F_{\xi_D}$$

$$F_\eta = F_{\eta_N} + F_{\eta_F} - F_{\eta_D}$$

$$F_\zeta = F_{\zeta_N} + F_{\zeta_F} - F_{\zeta_D}$$

### Gravity

$$G_x = \epsilon_1 \left(\frac{x}{|\vec{r}|^3}\right)$$

$$G_y = \epsilon_1 \left(\frac{y}{|\vec{r}|^3}\right)$$

$$G_z = \epsilon_2 \left(\frac{z}{|\vec{r}|^3}\right)$$

## Derivatives of Equations of Motion

Gravity (Cont'd)

$$\epsilon_1 = \frac{-G_m}{|\vec{r}|^2}\left[|\vec{r}|^2 + J\bar{A}^2\left(1 - \frac{5z^2}{|\vec{r}|^2}\right) - \frac{H\bar{A}^3 z}{|\vec{r}|^2}\left(-3 + \frac{7z^2}{|\vec{r}|^2}\right) + \frac{D\bar{A}^4}{|\vec{r}|^2}\left(\frac{9z^4}{|\vec{r}|^4} - \frac{6z^2}{|\vec{r}|^2} + \frac{3}{7}\right)\right]$$

$$\epsilon_2 = \frac{-G_m}{|\vec{r}|^2}\left[2J\bar{A}^2 - \frac{H\bar{A}^3 z}{|\vec{r}|^2}\left(-3 + \frac{3|\vec{r}|^2}{5z^2}\right) - \frac{D\bar{A}^4}{|\vec{r}|^2}\left(\frac{4z^2}{|\vec{r}|^2} - \frac{12}{7}\right)\right] + \epsilon_1$$

and J, H and D are constants. $\epsilon_1$ and $\epsilon_2$ optionally need not be recomputed for small changes in z position.

Acceleration

$$\dot{\vec{V}} = \frac{1}{M}\begin{bmatrix} \xi_x & \eta_x & \zeta_x \\ \xi_y & \eta_y & \zeta_y \\ \xi_z & \eta_z & \zeta_z \end{bmatrix}\begin{bmatrix} F_\xi \\ F_\eta \\ F_\zeta \end{bmatrix} + \vec{G}$$

or

$$\dot{\vec{V}} = -\vec{V}_a\left[\frac{F_{\xi_D}}{M|\vec{V}_a|}\right] + \vec{G}$$

in the case of spherical body drag. In this case ablation effects may be included:

$$M = M_I R_M$$

$$S = S_I R_S$$

where $R_M$ and $R_S$ are tabular mass and cross-sectional area ratio factors.

# Derivatives of Equations of Motion

## Integrals for Velocity, Position and Mass

$$\vec{V} = \int \dot{\vec{V}} \, dt$$

$$\vec{r} = \int \vec{V} \, dt$$

$$M = \int \dot{M} \, dt$$

## Turning Rate Options

Several computed turning rate options are available. In each option the rate computed ($\omega_g$) is limited to $\pm 2^{\circ}$ per integration cycle.

Zero lift flight (gravity turn)

Initially orient $\vec{V}$ to obtain $\alpha = 0$

$$\vec{V}_a = \left|\vec{V}_a\right| \vec{\xi}$$

then

$$\vec{V} = \Omega_e \begin{bmatrix} -y \\ x \\ 0 \end{bmatrix} + \vec{V}_a + \vec{V}_w$$

and compute pitch and yaw rates to maintain a constant attitude relationship between $\vec{\xi}$ and $\vec{V}_a$

$$\omega_{pg} = - \frac{\vec{\eta} \cdot \dot{\vec{V}}_a}{\vec{\xi} \cdot \vec{V}_a}$$

$$\omega_{yg} = \frac{\vec{\zeta} \cdot \dot{\vec{V}}_a}{\vec{\xi} \cdot \vec{V}_a}$$

## Derivatives of Equations of Motion

Turning Rate Options (Cont'd)

where

$$\dot{V}_{ax} = \dot{V}_x + \frac{x}{(x^2 + y^2)} \left[ (W_e)^2 + (V_{wz})^2 \right]$$

$$- \frac{VW2\ W_e}{(x^2 + y^2)^{1/2}} + 2\,\Omega_e\, V_{wy} + (\Omega_e)^2\, x$$

$$\dot{V}_{ay} = \dot{V}_y + \frac{y}{(x^2 + y^2)} \left[ (W_e)^2 + (V_{wz})^2 \right]$$

$$+ \frac{VW1\ W_e}{(x^2 + y^2)^{1/2}} - 2\Omega_e\, V_{wx} + (\Omega_e)^2\, y$$

$$\dot{V}_{az} = \dot{V}_z + \frac{z}{|\vec{r}|^2} (W_\phi)^2$$

and

$$VW1 = \frac{(W_\phi)\ x\ z}{|\vec{r}| \cdot (x^2 + y^2)^{1/2}}$$

$$VW2 = \frac{(W_\phi)\ y\ z}{|\vec{r}|\ (x^2 + y^2)^{1/2}}$$

## Derivatives of Equations of Motion

<u>Turning Rate Options</u> (Cont'd)

Roll to maintain pitch plane through center of Cartesian inertial system

$$\omega_{rg} = - \frac{\vec{V} \cdot \vec{\zeta}}{\vec{r} \cdot \vec{\eta}} + \frac{\vec{r} \cdot \vec{\xi}}{\vec{r} \cdot \vec{\eta}} \quad \omega_y$$

Note that $\omega_{rg}$ is indeterminant during vertical rise.

Instantaneous turns about pitch, yaw and/or roll axes (Fig. 2).

$$\vec{\xi} = \vec{\xi}_o \cos\theta_p - \vec{\eta}_o \sin\theta_p$$

$$\vec{\eta} = \vec{\xi}_o \sin\theta_p + \vec{\eta}_o \cos\theta_p$$

$$\vec{\zeta} = \vec{\zeta}_o$$

$$\vec{\xi} = \vec{\xi}_o \cos\theta_y + \vec{\zeta}_o \sin\theta_y$$

$$\vec{\eta} = \vec{\eta}_o$$

$$\vec{\zeta} = - \vec{\zeta}_o \sin\theta_y + \vec{\zeta}_o \cos\theta_y$$

$$\vec{\xi} = \vec{\xi}_o$$

$$\vec{\eta} = \vec{\eta}_o \cos\theta_r - \vec{\zeta}_o \sin\theta_r$$

$$\vec{\zeta} = \vec{\eta}_o \sin\theta_r + \vec{\zeta}_o \cos\theta_r$$

## Derivatives of Equations of Motion

Turning Rate Options (Cont'd)

Obtain a specified epsilon local ($\epsilon_{L_S}$)(see below) in a specified time ($\Delta t$) from the current $\epsilon_L(\epsilon_{L_C})$

$$\omega_{pg} = \frac{\epsilon_{L_S} - \epsilon_{L_C}}{\Delta t}$$

where

$\epsilon_L$ is the angle between $\vec{\xi}$ and $\vec{r}$. (Fig. 6)

Obtain $\epsilon_{L_S}$ immediately from $\epsilon_{L_C}$

$$\Psi = \epsilon_{L_S} - \epsilon_{L_C}$$

$$\vec{\xi} = \vec{\xi}_o \cos\Psi - \vec{\eta}_o \sin\Psi$$

$$\vec{\eta} = \vec{\xi}_o \sin\Psi + \vec{\eta}_o \cos\Psi$$

$$\vec{\zeta} = \vec{\zeta}_o$$

Make $\epsilon_L = \beta_I$, angle between $\vec{r}$ and $\vec{V}$ (Fig. 5)

$$\omega_{pg} = \frac{\beta_I - \epsilon_{L_C}}{\delta t} + \dot{\epsilon}_L$$

where $\delta t$ is the computing increment and

$$\dot{\epsilon}_L = \frac{|\vec{V}| \sin \beta_I}{|\vec{r}|}$$

## Derivatives of Equations of Motion

**Turning Rate Options** (Cont'd)

**Maintain a specified $\epsilon_L$**

$$\omega_{pg} = \frac{\epsilon_{L_M} - \epsilon_{L_C}}{\delta t} + \dot{\epsilon}_L$$

where $\epsilon_{L_M}$ is the $\epsilon_L$ to be maintained.

**Integrals for Attitude**

Integrands for attitude are

$$\dot{\vec{\xi}} = -\vec{\eta}\ \omega_p + \vec{\zeta}\ \omega_y$$

$$\dot{\vec{\zeta}} = -\vec{\xi}\ \omega_y + \vec{\eta}\ \omega_r$$

Attitudes are

$$\vec{\xi} = \int \dot{\vec{\xi}}\ dt$$

$$\vec{\zeta} = \int \dot{\vec{\zeta}}\ dt$$

$$\vec{\eta} = \vec{\zeta} \times \vec{\xi}$$

where

$$\omega_p = \omega_{pg} \text{ or } K_{\omega_{pt}}\ \omega_{pt} + \omega_{pc} + \omega_{pd}$$

$$\omega_y = \omega_{yg} \text{ or } K_{\omega_{yt}}\ \omega_{yt} + \omega_{yc} + \omega_{yd}$$

$$\omega_r = \omega_{rg} \text{ or } K_{\omega_{rt}}\ \omega_{rt} + \omega_{rc} + \omega_{rd}$$

## Auxiliary Computations

### Descriptive Quantities

These equations specify some descriptive quantities used for special purposes and/or print out.

Angle between $\vec{V}_a$ and $\vec{r}$ (Fig. 4)

$$\beta = \cos^{-1} \frac{\vec{r} \cdot \vec{V}_a}{|\vec{r}|\ |\vec{V}_a|} \qquad 0 \leq \beta \leq 180^\circ$$

$$\Gamma = \frac{\pi}{2} - \beta$$

Angle between $\vec{r}$ and $\vec{V}$ (Fig. 5)

$$\beta_I = \cos^{-1} \frac{\vec{r} \cdot \dot{\vec{r}}}{|\vec{r}|\ |\dot{\vec{r}}|} \qquad 0 \leq \beta_I \leq 180^\circ$$

$$\Gamma_I = \frac{\pi}{2} - \beta_I$$

Angle between $\vec{r}$ and $\vec{\xi}$ (Fig. 6)

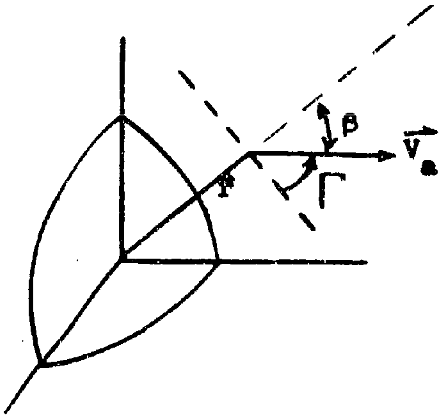
$$\epsilon_L = \cos^{-1} \frac{\vec{r} \cdot \vec{\xi}}{|\vec{r}|} \qquad 0 \leq \epsilon_L \leq 180^\circ$$

Angle between $\vec{\xi}$ and launch vertical ($\xi_L$) (Fig. 7)

$$\epsilon = \cos^{-1} \quad \vec{\xi} \cdot \vec{\xi}_L \qquad 0 \leq \epsilon \leq 180^\circ$$

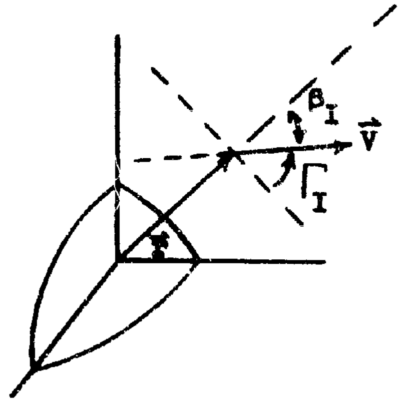
Azimuth from north of $\vec{\xi}$ projected on a plane normal to $\vec{r}$

$$A_{body} = \tan^{-1} \frac{|\vec{r}| \left[ x\ \xi_y - y\ \xi_x \right]}{y \left[ y\ \xi_z - z\ \xi_y \right] - x \left[ z\ \xi_x - x\ \xi_z \right]}$$



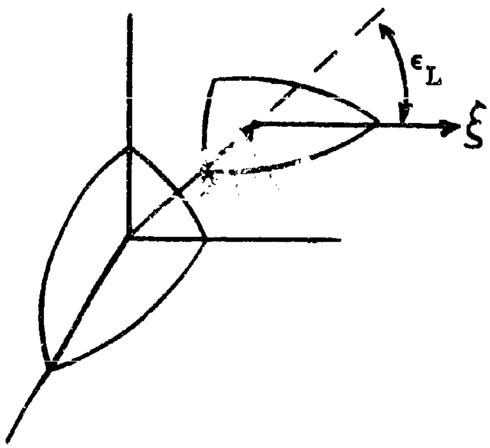

BETA

FIGURE 4



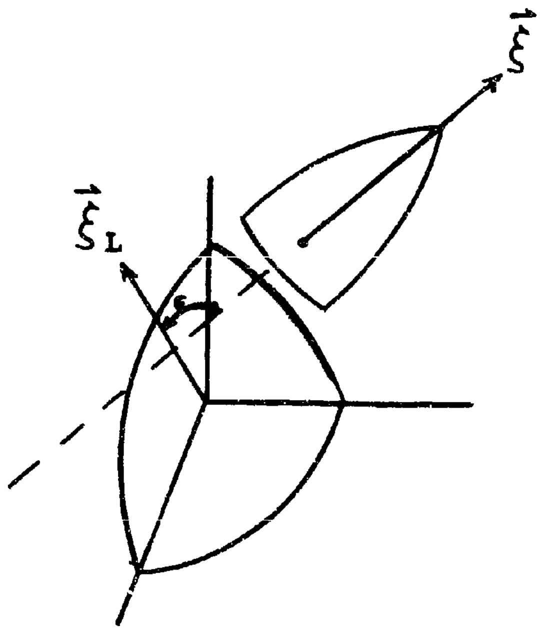

BETI

FIGURE 5




EPSILON LOCAL

FIGURE 6




EPSILON

FIGURE 7

Auxiliary Computations

Descri tive Quantities (Cont'd)

Normalized angular momentum

$$RVSB = |\vec{r}| \; |\vec{V}| \; \sin \beta_I$$

Ratio squared of velocity to circular orbital velocity

$$RVSG = \frac{|\vec{V}|^2 \; |\vec{r}|}{G_m}$$

Total energy

$$ENGY = \frac{|\vec{V}|^2}{2} - \frac{G_m}{|\vec{r}|}$$

Vis viva energy

$$VVEN = 2 \; ENGY$$

Sensed velocity

$$V_S = \int (\dot{\vec{V}} - \vec{G}) \, dt$$

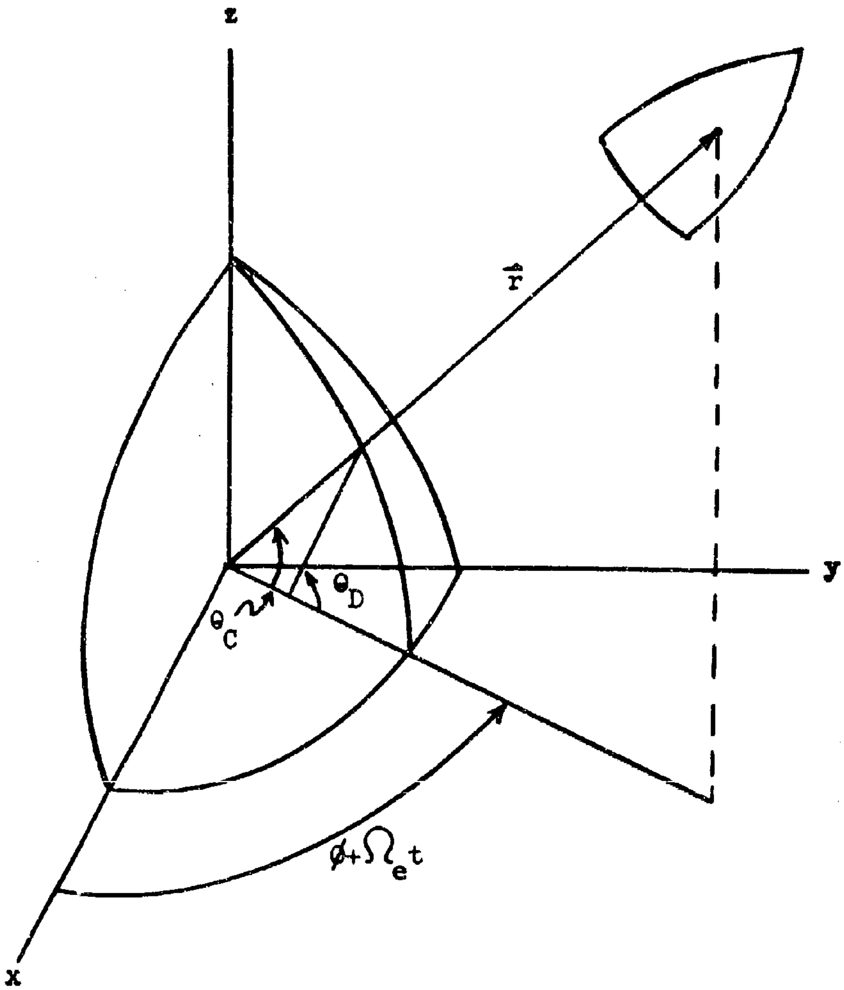
Aerodynamic heating

$$H_A = \int QV_a \, dt$$

Geodetic latitude ($\Theta_D$) and longitude ($\phi$)(Fig. 8)

$$\Theta_D = \tan^{-1} \quad \frac{z}{(1-e)^2 (x^2 + y^2)^{1/2}}$$

$$\phi = \left[ \tan^{-1} \left( \frac{y}{x} \right) \right] - \Omega_e \, t$$




LATITUDE ∉ LONGITUDE

FIGURE 8

## Auxiliary Computations

Descriptive Quantities (Cont'd)

$$\theta_c = \tan^{-1}\left[\frac{z}{(x^2 + y^2)^{1/2}}\right]$$

Circular range (R) between two positions on the spheroid, where $(\theta_{c1}, \phi_1)$ (geocentric latitude, longitude) defines the first position, $(\theta_{c2}, \phi_2)$ defines the second, and $r_o$ is the radius of the spheroid, is given by

$$E_\Psi = \sin\theta_{c1} \sin\theta_{c2} + \cos\theta_{c1} \cos\theta_{c2} \cos(\phi_2 - \phi_1)$$

If

$$E_\Psi \geq .99998, \text{ then}$$

$$\Psi = \left[(\theta_{c2} - \theta_{c1})^2 + (\phi_2 - \phi_1)^2 \cos^2\theta_{c1}\right]^{1/2}$$

where

$$\left|\theta_{c2} - \theta_{c1}\right| \text{ and } \left|\phi_2 - \phi_1\right| \text{ are} \leq 180^\circ$$

If

$$E_\Psi < .99998, \text{ then}$$

$$\Psi = \cos^{-1} E_\Psi$$

$$R = \begin{cases} 2\pi - \Psi, & \text{if } 90^\circ \leq |p| \leq 180^\circ \\ \Psi, & \text{if } 0^\circ \leq |p| < 90^\circ \end{cases}$$

where p is the clockwise difference of the azimuth from $(\theta_{c1}, \phi_1)$ to $(\theta_{c2}, \phi_2)$ and a reference azimuth.

## Auxiliary Computations

**Descriptive Quantities** (Cont'd)

Azimuth from North of $\vec{V}$ projected on a plane normal to $\vec{r}$ is

$$A_V = \tan^{-1}\left[\frac{\dot{y}\cos(\Omega_e t + \phi) - \dot{x}\sin(\Omega_e t + \phi)}{\dot{z}\cos\theta_c - \sin\theta_c\left[\dot{x}\cos(\Omega_e t + \phi) + \dot{y}\sin(\Omega_e t + \phi)\right]}\right]$$

Cartesian inertial to launch centered inertial coordinate transformation

$$\begin{bmatrix} p_{xg} \\ p_{yg} \\ p_{zg} \end{bmatrix} = \begin{bmatrix} -\eta_{x_o} & -\eta_{y_o} & -\eta_{z_o} \\ -\zeta_{x_o} & -\zeta_{y_o} & -\zeta_{z_o} \\ -\xi_{x_o} & -\xi_{y_o} & -\xi_{z_o} \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} - \begin{bmatrix} -\eta_{x_o} & -\eta_{y_o} & -\eta_{z_o} \\ -\zeta_{x_o} & -\zeta_{y_o} & -\zeta_{z_o} \\ -\xi_{x_o} & -\xi_{y_o} & -\xi_{z_o} \end{bmatrix} \begin{bmatrix} x_o \\ y_o \\ z_o \end{bmatrix}$$

$$\begin{bmatrix} v_{xg} \\ v_{yg} \\ v_{zg} \end{bmatrix} = \begin{bmatrix} -\eta_{x_o} & -\eta_{y_o} & -\eta_{z_o} \\ -\zeta_{x_o} & -\zeta_{y_o} & -\zeta_{z_o} \\ -\xi_{x_o} & -\xi_{y_o} & -\xi_{z_c} \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{bmatrix}$$

where $\vec{\xi}_o$, $\vec{\eta}_o$, $\vec{\zeta}_o$ are the attitude vectors at launch.

**Miss Distances**

Cross range miss distance ($M_c$) and down range miss distance ($M_d$) components of total miss distance ($M_t$) at impact (Fig. 9) are given by

$$E_{L,} \cdot \quad \left[\sin\theta_{CL}\sin\theta_{CI} + \cos\theta_{CL}\cos\theta_{CI}\cos(\phi_i - \phi_L)\right]$$

## Auxiliary Computations

**Miss Distances** (Cont'd)

If

$E_{L,I} \geq .99998$, then

$$\Psi_{L,I} = \left[(\theta_{CI} - \theta_{CL})^2 + (\phi_I - \phi_L)^2 \cos^2\theta_{CL}\right]^{1/2}$$

where

$$\left|\theta_{CI} - \theta_{CL}\right| \text{ and } \left|\phi_I - \phi_L\right| \text{ are } \leq 180^\circ$$

If

$E_{L,I} < .99998$, then

$$\Psi_{L,I} = \cos^{-1} E_{L,I}; \quad 0 \leq \Psi_{L,I} \leq 180^\circ$$

$$E_{L,M} = \sin\theta_{CL} \sin\theta_{CM} + \cos\theta_{CL} \cos\theta_{CM} \cos(\phi_M - \phi_L)$$

If

$E_{L,M} \geq .99998$, then

$$\Psi_{L,M} = \left[(\theta_{CM} - \theta_{CL})^2 + (\phi_M - \phi_L)^2 \cos^2\theta_{CL}\right]^{1/2}$$

where

$$\left|\theta_{CM} - \theta_{CL}\right| \text{ and } \left|\phi_M - \phi_L\right| \text{ are } \leq 180^\circ$$

If

$E_{L,M} < .99998$, then

$$\Psi_{L,M} = \cos^{-1} E_{L,M}; \quad 0 \leq \Psi_{L,M} \leq 180^\circ$$

## Auxiliary Computations

**Miss Distances** (Cont'd)

$$\sin\eta_{L,I} = \frac{\cos\Theta_{CI}\,\sin(\phi_I - \phi_L)}{\sin\Psi_{L,I}}$$

$$\cos\eta_{L,I} = \frac{(\sin\Theta_{CL}\,\cos\Psi_{L,I}) - \sin\Theta_{CI}}{\cos\Theta_{CL}\,\sin\Psi_{L,I}}$$

$$\sin\eta_{L,M} = \frac{\cos\Theta_{CM}\,\sin(\phi_M - \phi_L)}{\sin\Psi_{L,M}}$$

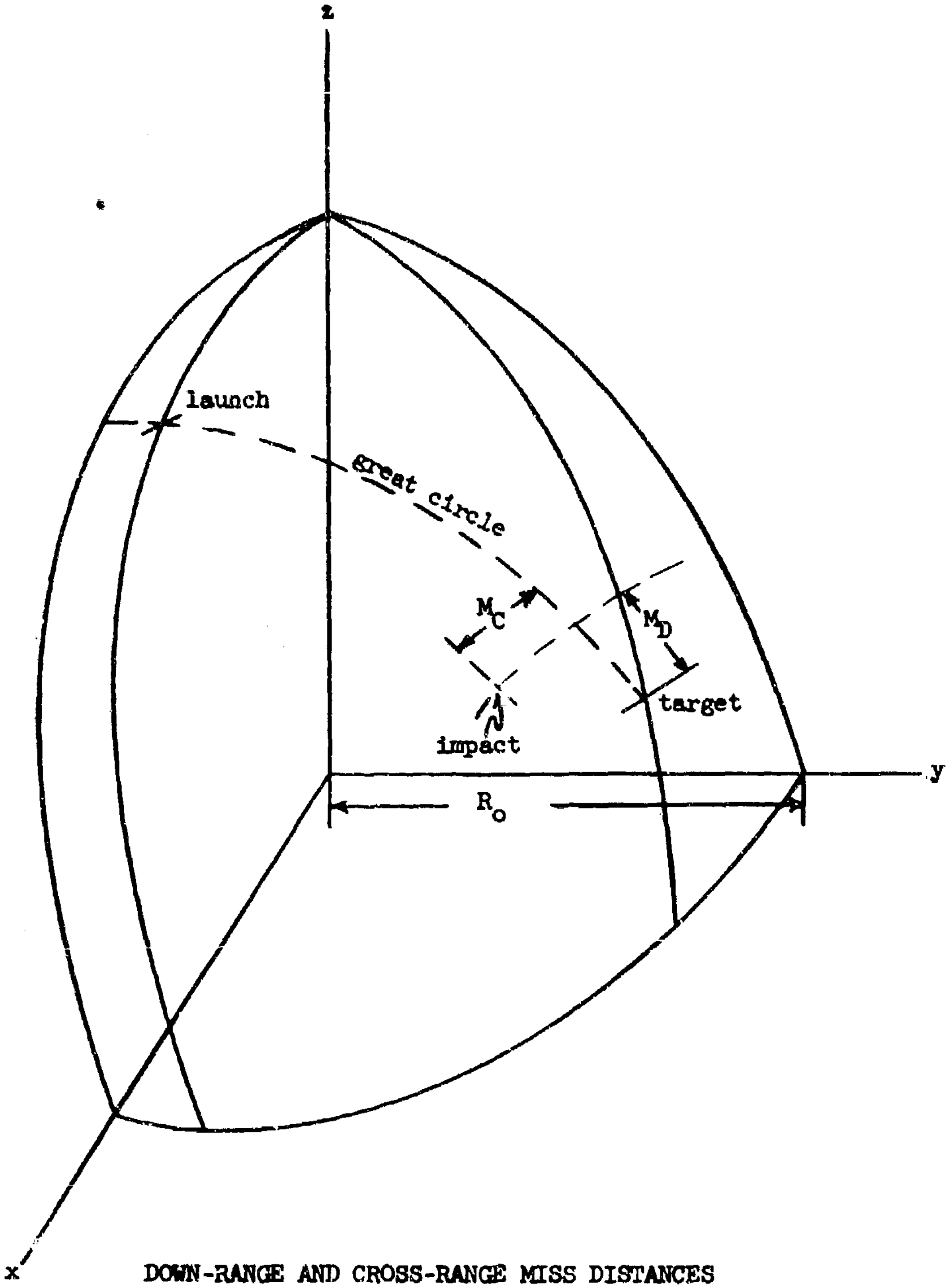
$$\cos\eta_{L,M} = \frac{(\sin\Theta_{CL}\,\cos\Psi_{L,M}) - \sin\Theta_{CM}}{\cos\Theta_{CL}\,\sin\Psi_{L,M}}$$

$$\sin M_c' = (\sin\eta_{L,M}\,\cos\eta_{L,I} - \cos\eta_{L,M}\,\sin\eta_{L,I})\sin\Psi_{L,M}$$

$$\cos M_c' = \left[1 - \sin^2 M_c'\right]^{1/2}$$

$$(\Psi_{L,I} + M_d') = \cos^{-1}\left[\frac{\cos\Psi_{L,M}}{\cos M_c'}\right]$$

$$M_d = r_o\left[(\Psi_{L,I} + M_d') - \Psi_{L,I}\right]$$




DOWN-RANGE AND CROSS-RANGE MISS DISTANCES

FIGURE 9

Mis

Orb:

## Auxiliary Computations

**Miss Distances** (Cont'd)

$$M_c = r_o M_c'$$

$$M_t = \left[ M_c^{\,2} + M_d^{\,2} \right]^{1/2}$$

where $M_c'$, $M_d'$ are central angles. Note that $M_c$ is positive to the left when looking down range.

**Orbital and Impact Approximations**

For an elliptical orbit or an impact trajectory,

$$\epsilon \cos\bar{E} = \frac{\left|\vec{r}\right| \cdot \left|\vec{V}\right|^2}{G_M} - 1$$

$$\bar{A} = \frac{\left|\vec{r}\right|}{1 - \epsilon \cos\bar{E}}$$

$$\epsilon \sin\bar{E} = \frac{\vec{r} \cdot \vec{V}}{\sqrt{\bar{A} \cdot G_M}}$$

$$\epsilon = \left[ (\epsilon \cos\bar{E})^2 + (\epsilon \sin\bar{E})^2 \right]^{1/2}$$

$$\text{Perigee} = \bar{A}(1 - \epsilon) - r_o$$

$$\text{Apogee} = \bar{A}(1 + \epsilon) - r_o$$

where $\epsilon$ is the ellipsoidal eccentricity and $\bar{E}$ is the eccentric anomaly of the present missile position.

## Auxiliary Computations

Orbital and Impact Approximations (Cont'd)

$$\text{Period} = \frac{2\pi\, \bar{A}^{3/2}}{60\sqrt{G_M}}$$

$$\text{Inclination} = \cos^{-1}\left(\sin A_V \cos\theta_{CM}\right)$$

$$\text{Velocity at apogee} = \frac{\left[\left|\vec{r}\right| \left|\vec{V}\right| \left(1 - \cos^2\beta_I\right)\right]^{1/2}}{\bar{A}(1 + \epsilon)}$$

For an impact trajectory $R_{SEI} \geq \bar{A}(1 - e)$

$$\epsilon \cos E_T = 1 - \frac{R_{SEI}}{\bar{A}}$$

$$\epsilon \sin E_T = -\left[\epsilon^2 - (\epsilon \cos E_T)^2\right]^{1/2}$$

$$\sin(E_T - \bar{E}) = \frac{(\epsilon \sin E_T)(\epsilon \cos\bar{E}) - (\epsilon \cos E_T)(\epsilon \sin\bar{E})}{\epsilon^2}$$

$$\cos(E_T - \bar{E}) = \frac{(\epsilon \cos E_T)(\epsilon \cos\bar{E}) + (\epsilon \sin E_T)(\epsilon \sin\bar{E})}{\epsilon^2}$$

$$E_T - \bar{E} = \tan^{-1}\left[\frac{\sin(E_T - \bar{E})}{\cos(E_T - \bar{E})}\right]$$

where $E_T$ is the eccentric anomaly at the impact point.

Auxiliary Computations

Orbital and Impact Approximations (Cont'd)

Time of flight to impact and time of impact are

$$T_F = \frac{\bar{A}^{3/2}}{\sqrt{G_M}} \left[ E_T - \bar{E} + \epsilon \sin\bar{E} - \epsilon \sin E_T \right]$$

$$T_{IMP} = t + T_F$$

Inertial Cartesian components of impact point are

$$x_{SEI} = \dot{x}f + xg$$

$$y_{SEI} = \dot{y}f + yg$$

$$z_{SEI} = \dot{z}f + zg$$

where

$$f = \frac{\bar{A}^{3/2}}{\sqrt{G_M}} \left[ \sin(E_T - \bar{E}) + \epsilon \sin\bar{E} - \epsilon \sin E_T \right]$$

$$g = \frac{\cos(E_T - \bar{E}) - \epsilon \cos\bar{E}}{1 - \epsilon \cos\bar{E}}$$

Azimuth of impact is

$$A_{z_I} = - \left[ \tan^{-1} \frac{\sin(180 - A_{z_I})}{\cos(180 - A_{z_I})} - 180 \right]$$

Auxiliary Computations

Orbital and Impact Approximations (Cont'd)

where

$$\sin(180 - A_{z_I}) = \frac{\cos\Theta_{CI}\,\sin(\phi_I - \phi_L)}{\sin\Psi}$$

$$\cos(180 - A_{z_I}) = \frac{\sin\Theta_{CL}\cos\Psi - \sin\Theta_{CI}}{\cos\Theta_{CL}\sin\Psi}$$

$$E_\Psi = \sin\Theta_{CL}\sin\Theta_{CI} + \cos\Theta_{CL}\cos\Theta_{CI}\cos(\phi_L - \phi_I)$$

If

$$E_\Psi \geq .99998, \text{ then } \Psi = \left[(\Theta_{CI} - \Theta_{CL})^2 + (\phi_I - \phi_L)^2\cos^2\Theta_{CL}\right]^{1/2}$$

where

$$\left|\Theta_{CI} - \Theta_{CL}\right| \text{ and } \left|\phi_I - \phi_L\right| \text{ are} \leq 180^\circ$$

If

$$E_\Psi < .99998,\ \Psi = \cos^{-1}E_\Psi,\ 0 \leq \Psi \leq 180^\circ$$

Circular range to impact is

$$CRIP = \begin{cases} 2\pi - \Psi; & \text{if } 90^\circ \leq |p| \leq 180^\circ \\ \Psi; & \text{if } 0^\circ \leq |p| < 90^\circ \end{cases}$$

where p is the clockwise difference of impact azimuth and launch azimuth.

## Auxiliary Computations

### Orbital and Impact Approximations (Cont'd)

If $|\vec{r}| < R_{SEI}$ or if the missile is in a hyperbolic orbit (i.e.

$\frac{|\vec{r}|\ |\vec{V}|^2}{G_M} > 2$), all orbital and impact quantities are set to zero.

$R_{SEI}$ (radius of spherical earth impact) may be input directly or as a height relative to the radius of the target $\left[R_{SEI} = f(\phi_T, \Theta_T, H_T, EFHT)\right]$ or as a height relative to the average radius of the geoid ($r_o$).

### Velocity Losses and Impulse Calculations

Gravity loss

$$V_g = \int_{t_1}^{t_2} g \cos\beta_I \, dt$$

where

$t_1$ = time at beginning of stage

$t_2$ = time at end of stage

g = local gravity

$\beta_I$ = angle between $\vec{r}$ and geocentric local vertical ($\vec{r}$)

Drag loss

$$V_d = \int_{t_1}^{t_2} \frac{{}^F\xi_D}{M} \, dt$$

Misalignment loss

$$V_m = \int_{t_1}^{t_2} \frac{{}^F\xi_F}{M} (1 - \cos\alpha) \, dt$$

Auxiliary Computations

Velocity Losses and Impulse Calculations (Cont'd)

Average specific impulse per stage

$$\bar{I}_{sp} = -\left[\frac{1}{t_2 - t_1}\right] \int_{t_1}^{t_2} \frac{{}^{F}\xi_{F}}{\dot{M}}\, dt$$

Total impulse

$$I_T = \int_{t_1}^{t_2} {}^{F}\xi_{F} \quad dt$$

Ideal velocity per stage

$$V_I = g_o \, \bar{I}_{sp} \, \ell n \left(\frac{wt_1}{wt_2}\right)$$

where $g_o$ is the nominal value of gravity, $wt_1$ is the weight of missile at $t_1$, and $wt_2$ is the weight of missile at $t_2$.

# Radar Conversions

## Radar Site Computations

Distance from radar site to polar axis is

$$x_{ER} = \left[ r_{SL_R} \cos\theta_{CR} + (h_R + g_{SR}) \cos\theta_{DR} \right]$$

where

$$r_{SL_R} \cos\theta_{CR} = \frac{\bar{A}}{\left[1 + (1-e)^2 \tan^2\theta_{DR}\right]^{1/2}}$$

and

$h_R$ = altitude of radar site above sea level

$g_{SR}$ = geoidal separation at radar site

$r_{SL_R}$ = sea level radius at radar site

$\delta\phi_R$ = longitude of radar, positive east

Distance of radar site above equatorial plane is

$$z_{ER} = r_{SL_R} \sin\theta_{CR} + (h_R + g_{SR}) \sin\theta_{DR}$$

where

$$r_{SL_R} \sin\theta_{CR} = (r_{SL_R} \cos\theta_{CR})(1-e)^2 \tan\theta_{DR}$$

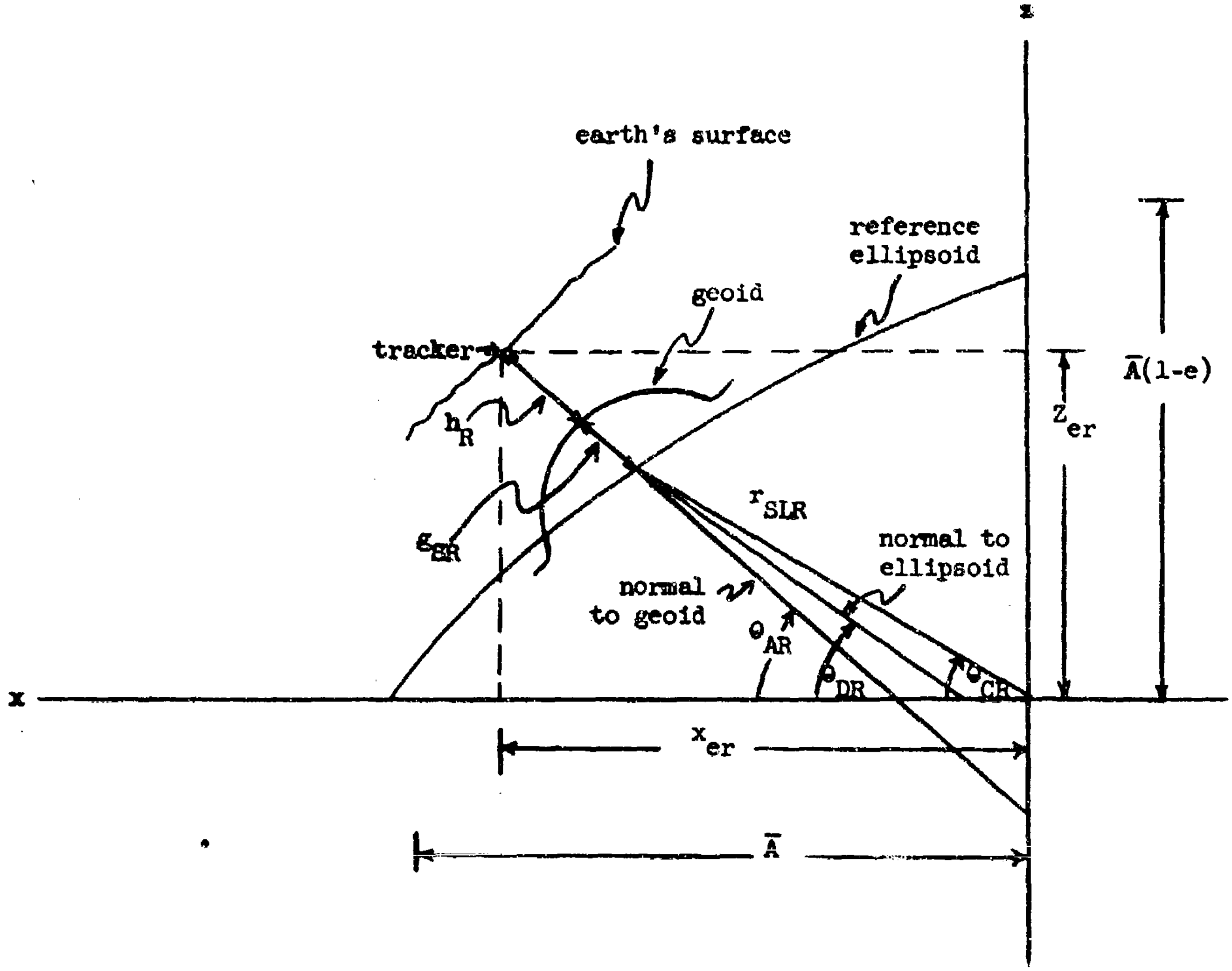
## Site Centered Cartesian

Ellipsoid centered cartesian inertial coordinates converted to ellipsoid-fixed rectangular with origin at radar (Fig. 10) are given by the following:

$$x_e = x \cos\Psi + y \sin\Psi$$

$$y_e = -x \sin\Psi + y \cos\Psi$$

$$z_e = z$$




TRACKER COORDINATES ON REFERENCE ELLIPSOID

FIGURE 10

<u>Site Centered Cartesian</u> (Cont'd)

where

$$\Psi = \delta\phi_R + \Omega_e t$$

$$\bar{x}_e = (x_e - x_{e_R})\sin\theta_{AR} - (z_e - z_{eR})\cos\theta_{AR}$$

$$\bar{y}_e = y_e$$

$$\bar{z}_e = (x_e - x_{e_R})\cos\theta_{AR} + (z_e - z_{e_R})\sin\theta_{AR}$$

$$\bar{u} = \bar{x}_e$$

$$\bar{v} = \bar{y}_e + \delta_{w_R} \cdot \bar{z}_e$$

$$\bar{w} = -\delta_{w_R} \cdot \bar{y}_e + \bar{z}_e$$

where $\delta_{w_R}$ = deviation from local vertical, and $\sin\delta_w \doteq \delta_w$

$$u = \bar{u}\cos\mu_R + \bar{v}\sin\mu_R$$

$$v = -\bar{u}\sin\mu_R + \bar{v}\cos\mu_R$$

$$w = \bar{w}$$

where $\mu_R$ = angle through which the $\bar{u} - \bar{v}$ plane is rotated to fix radar azimuth reference

$$\dot{x}_e = \dot{x}\cos\Psi + \dot{y}\sin\Psi + \Omega_e \cdot y_e$$

$$\dot{y}_e = -\dot{x}\sin\Psi + \dot{y}\cos\Psi - \Omega_e \cdot x_e$$

$$\dot{z}_e = \dot{z}$$

## Radar Conversions

Site Centered Cartesian (Cont'd)

$$\bar{\dot{x}}_e = \dot{x}_e \sin\theta_{AR} - \dot{z}_e \cos\theta_{AR}$$

$$\bar{\dot{y}}_e = \dot{y}_e$$

$$\bar{\dot{z}}_e = \dot{x}_e \cos\theta_{AR} + \dot{z}_e \sin\theta_{AR}$$

$$\bar{\dot{u}} = \bar{\dot{x}}_e$$

$$\bar{\dot{v}} = \bar{\dot{y}}_e + \delta_W \cdot \bar{\dot{z}}_e$$

$$\bar{\dot{w}} = -\delta_W \cdot \bar{\dot{y}}_e + \bar{\dot{z}}_e$$

$$\dot{u} = \bar{\dot{u}} \cos\mu_R + \bar{\dot{v}} \sin\mu_R$$

$$\dot{v} = -\bar{\dot{u}} \sin\mu_R + \bar{\dot{v}} \cos\mu_R$$

$$\dot{w} = \bar{\dot{w}}$$

Spherical Radar

Ellipsoid-fixed rectangular coordinates converted to spherical (radar) coordinates (Fig. 12)

$$R = \left[u^2 + v^2 + w^2\right]^{1/2}$$

**Radar Conversions**

**Spherical Radar** (Cont'd)

$$A = \sin^{-1}\left[\frac{u}{(u^2 + v^2)^{1/2}}\right] = \tan^{-1}\left(\frac{u}{v}\right), \quad 0 \leqslant A \leqslant 2\pi$$

$$E = \cos^{-1}\left(\frac{w}{R}\right), \quad 0 \leqslant E \leqslant \pi$$

$$\dot{R} = \frac{1}{R}(u\dot{u} + v\dot{v} + w\dot{w})$$

$$\dot{A} = \frac{v\dot{u} - u\dot{v}}{(u^2 + v^2)}$$

$$\dot{E} = \frac{w(u\dot{u} + v\dot{v}) - \dot{w}(u^2 + v^2)}{R^2(u^2 + v^2)^{1/2}}$$

Doppler Radar

Radar cross-shaped doppler rates computed from ellipsoid-fixed rectangular coordinates (Fig. 13)

$$\dot{p} = \frac{\dot{u}}{R}\left\{u\left[(\delta_1 - \delta_3) + \frac{3}{2}(\delta_1{}^2 - \delta_3{}^2) + \frac{5}{2}(\delta_1{}^3 - \delta_3{}^3)\right]\right.$$

$$\left. + u_3\left[1 + \delta_3 + \frac{3}{2}\delta_3{}^2 + \frac{5}{2}\delta_3{}^3\right] - u_1\left[1 + \delta_1 + \frac{3}{2}\delta_1{}^2 + \frac{5}{2}\delta_1{}^3\right]\right\}$$

Radar Conversions

<u>Doppler Radar</u> (Cont'd)

$$+ \frac{\dot{v}}{R} \left\{ v \left[ (\delta_1 - \delta_3) + \frac{3}{2}(\delta_1^2 - \delta_3^2) + \frac{5}{2}(\delta_1^3 - \delta_3^3) \right] \right.$$

$$\left. = v_3 \left[ 1 + \delta_3 + \frac{3}{2}\delta_3^2 + \frac{5}{2}\delta_3^3 \right] - v_1 \left[ 1 + \delta_1 + \frac{3}{2}\delta_1^2 + \frac{5}{2}\delta_1^3 \right] \right\}$$

$$+ \frac{\dot{w}}{R} \left\{ w \left[ (\delta_1 - \delta_3) + \frac{3}{2}(\delta_1^2 - \delta_3^2) + \frac{5}{2}(\delta_1^3 - \delta_3^3) \right] \right.$$

$$\left. + w_3 \left[ 1 + \delta_3 + \frac{3}{2}\delta_3^2 + \frac{5}{2}\delta_3^3 \right] - w_1 \left[ 1 + \delta_1 + \frac{3}{2}\delta_1^2 + \frac{5}{2}\delta_1^3 \right] \right\}$$

$$\dot{q} = \frac{\dot{u}}{R} \left\{ u \left[ (\delta_2 - \delta_4) + \frac{3}{2}(\delta_2^2 - \delta_4^2) + \frac{5}{2}(\delta_2^3 - \delta_4^3) \right] \right.$$

$$\left. + u_4 \left[ 1 + \delta_4 + \frac{3}{2}\delta_4^2 + \frac{5}{2}\delta_4^3 \right] - u_2 \left[ 1 + \delta_2 + \frac{3}{2}\delta_2^2 + \frac{5}{2}\delta_2^3 \right] \right\}$$

$$+ \frac{\dot{v}}{R} \left\{ v \left[ (\delta_2 - \delta_4) + \frac{3}{2}(\delta_2^2 - \delta_4^2) + \frac{5}{2}(\delta_2^3 - \delta_4^3) \right] \right.$$

$$\left. + v_4 \left[ 1 + \delta_4 + \frac{3}{2}\delta_4^2 + \frac{5}{2}\delta_4^3 \right] - v_2 \left[ 1 + \delta_2 + \frac{3}{2}\delta_2^2 + \frac{5}{2}\delta_2^3 \right] \right\}$$

## Radar Conversions

<u>Doppler Radar</u> (Cont'd)

$$+ \frac{\dot{w}}{R} \left\{ w \left[ (\delta_2 - \delta_4) + \frac{3}{2} (\delta_2^{\,2} - \delta_4^{\,2}) + \frac{5}{2} (\delta_2^{\,3} - \delta_4^{\,3}) \right] \right.$$

$$\left. + w_4 \left[ 1 + \delta_4 + \frac{3}{2} \delta_4^{\,2} + \frac{5}{2} \delta_4^{\,3} \right] - w_2 \left[ 1 + \delta_2 + \frac{3}{2} \delta_2^{\,2} + \frac{5}{2} \delta_2^{\,3} \right] \right\}$$

where

$$\delta_i = \frac{(uu_i + vv_i + ww_i) - \frac{1}{2} (u_i^{\,2} + v_i^{\,2} + w_i^{\,2})}{R^2}$$

Redar L-shaped doppler position and rates computed from ellipsoid-fixed coordinates (Fig. 13) are given by the following. Constants are delineated on Page 52.

$$R^2 = u^2 + v^2 + w^2$$

$$R_o = \left[ R^2 + K_4 - uK_1 - vK_2 - wK_3 \right]^{1/2}$$

$$R_1 = \left[ R^2 + K_8 - uK_5 - vK_6 - wK_7 \right]^{1/2}$$

$$R_2 = \left[ R^2 + K_{12} - uK_9 - vK_{10} - wK_{11} \right]^{1/2}$$

$$p = \frac{K_{16} + 2uK_{13} + 2vK_{14} + 2wK_{15}}{R_o + R_1}$$

Radar Conversions

Doppler Radar (Cont'd)

$$q = \frac{K_{20} + 2uK_{17} + 2vK_{18} + 2wK_{19}}{R_o + R_2}$$

$$\dot{R}_o = \frac{\dot{u}(u - K_{24}) + \dot{v}(v - K_{25}) + \dot{w}(w - K_{26})}{R_o}$$

$$\dot{p} = \frac{\dot{u}K_{13} + \dot{v}K_{14} + \dot{w}K_{15} - p\,\dot{R}_o}{R_1}$$

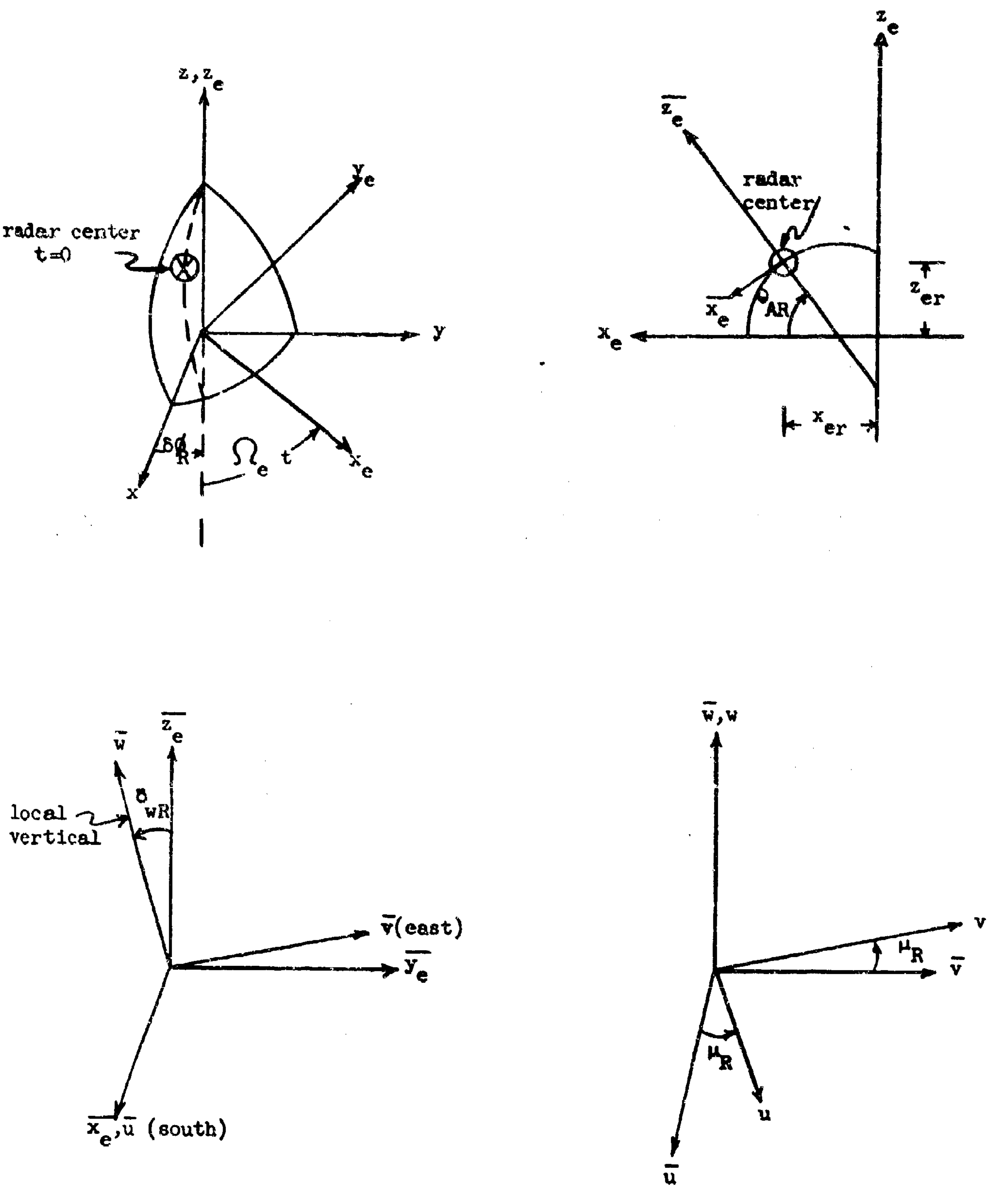
$$\dot{q} = \frac{\dot{u}K_{17} + \dot{v}K_{18} + \dot{w}K_{19} - q\,\dot{R}_o}{R_2}$$

Look Angles

Missile look angles from radar site are

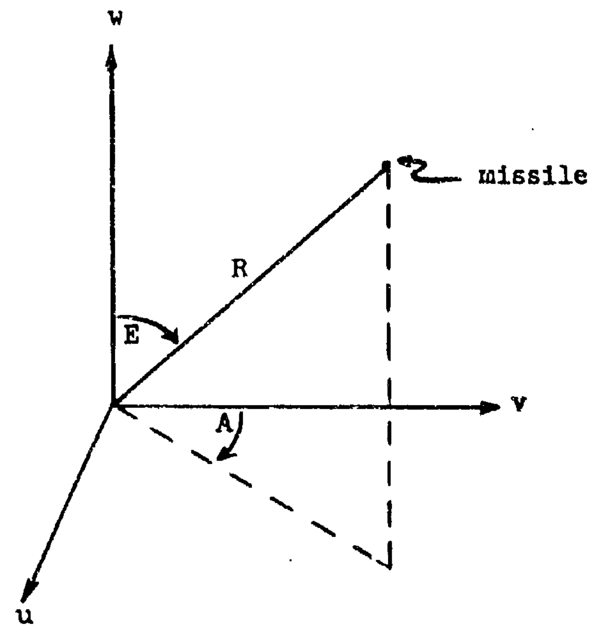
$$RLA1 = \cos^{-1}\left[\frac{(x - x_r)\,\xi_x + (y - y_r)\,\xi_y + (z - z_r)\,\xi_z}{\left[(x - x_r)^2 + (y - y_r)^2 + (z - z_r)^2\right]^{1/2}}\right]$$

$$RLA2 = \tan^{-1}\left[\frac{(x - x_r)\,\zeta_x + (y - y_r)\,\zeta_y + (z - z_r)\,\zeta_z}{(x - x_r)\,\eta_x + (y - y_r)\,\eta_y + (z - z_r)\,\eta_z}\right]$$



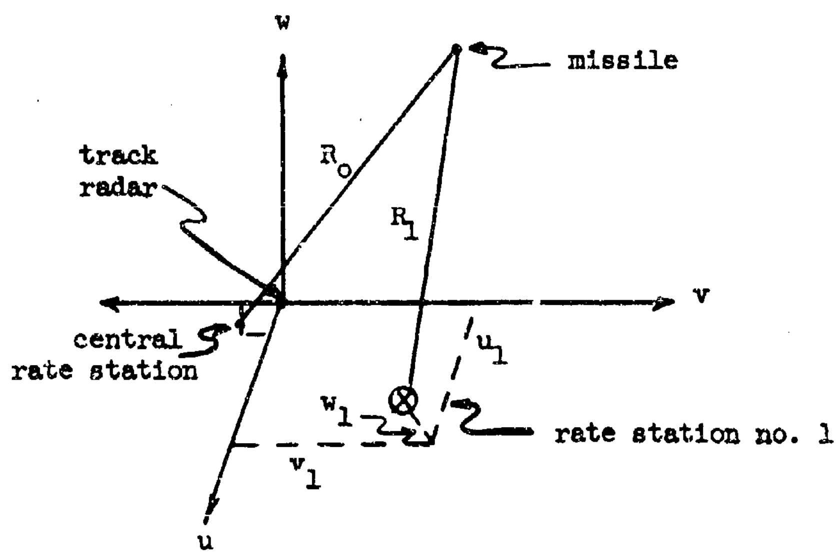

COORDINATE TRANSFORMATIONS

FIGURE 11




SITE - FIXED RECTANGULAR TO SPHERICAL CONVERSION

FIGURE 12




DOPPLER RATE STATION SETUP

FIGURE 13

Radar Conversions

Look Angles (Cont'd)

where

$$x_r = x_{er} \cos(\Omega_e t + \phi_r)$$

$$y_r = x_{er} \sin(\Omega_e t + \phi_r)$$

$$z_r = z_{er}$$

Pitch and yaw look angles

If RLA1 = $90^\circ$ or $270^\circ$

LAY = $90^\circ$ with opposite sign of RLA2

If $\left|RLA2\right| < 90^\circ$; LAP = $-90^\circ$

If $\left|RLA2\right| \geq 90^\circ$; LAP = $90^\circ$

If RLA1 $\neq 90^\circ$ or $270^\circ$

$$\tan RLA1 = \frac{\sin RLA1}{\cos RLA1}$$

$$\overline{LAP} = \tan^{-1}\left[-(\cos RLA2)(\tan RLA1)\right]$$

$$\overline{LAY} = \tan^{-1}\left[-(\sin RLA2)(\tan RLA1)\right]$$

If RLA1 $\leqslant 90^\circ$

$$LAP = \overline{LAP}$$

$$LAY = \overline{LAY}$$

If RLA1 $> 90^\circ$

If RLA2 $\leqslant -90^\circ$

$$LAP = \left|\overline{LAP}\right|$$

$$LAY = \overline{LAY} + 180^\circ$$

Radar Conversions

Look Angles (Cont'd)

If RLA2 $> -90^\circ$

If RLA2 $\leq 0^\circ$

$$LAP = -\left|\overline{LAP}\right|$$

$$LAY = \overline{LAY} + 180^\circ$$

If RLA2 $> 0^\circ$

If RLA2 $\leq 90^\circ$

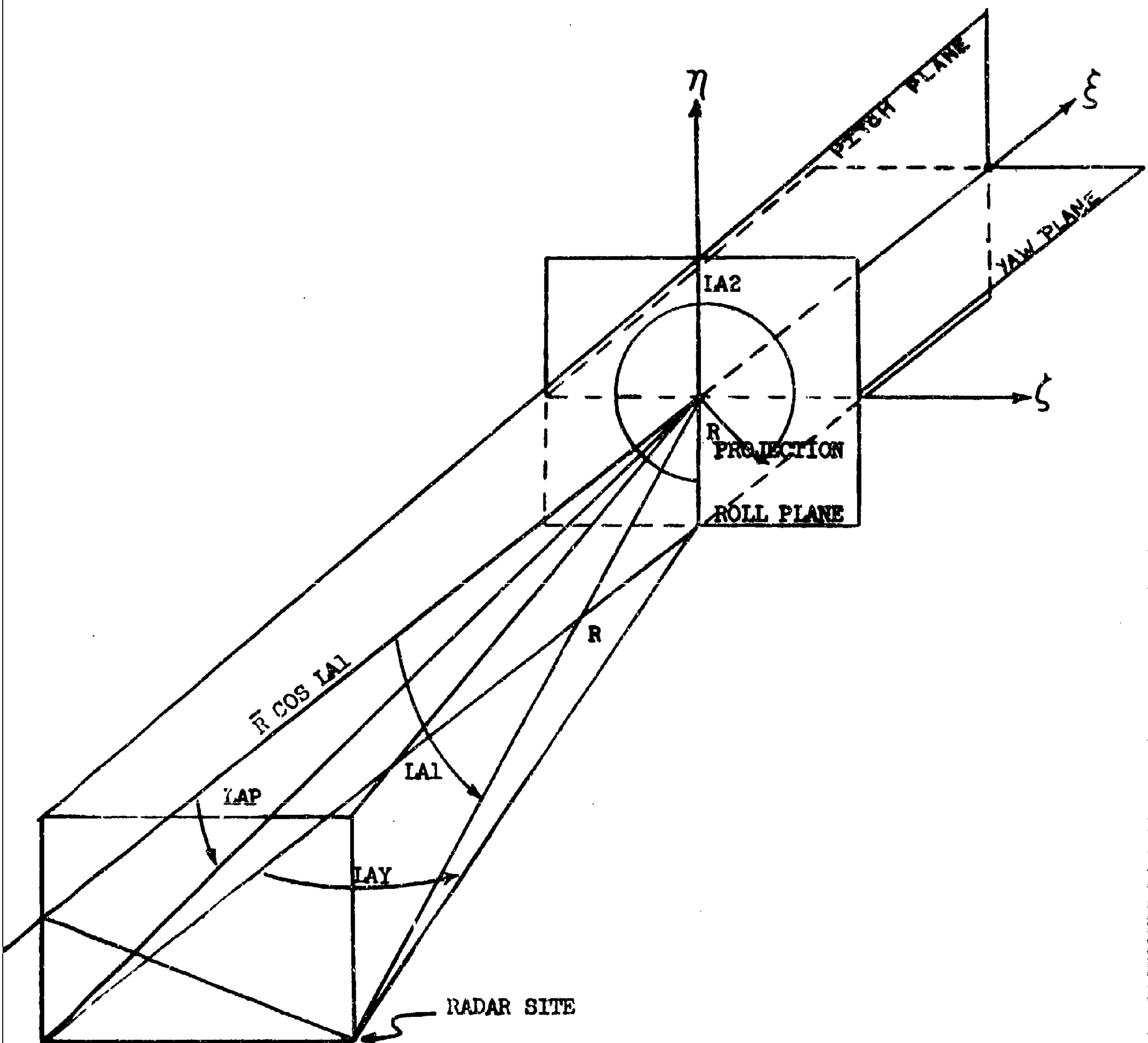
$$LAP = -\left|\overline{LAP}\right|$$

$$LAY = \overline{LAY} - 180^\circ$$

If RLA2 $> 90^\circ$

$$LAP = \left|\overline{LAP}\right|$$

$$LAY = \overline{LAY} - 180^\circ$$




RADAR LOOK ANGLES

FIGURE 14

## Initialization

### Launch Azimuth Estimation

$$T_{F_i} = \begin{cases} 1800, i = 0 \\ K_o + K_1\theta_{i-1} + K_2\theta^2_{i-1}, \; i > 0 \end{cases}$$

$$\theta_i = \cos^{-1}\left[\sin\theta_{CL}\sin\theta_{CT} + \cos\theta_{CL}\cos\theta_{CT}\cos(\phi_T - \phi_L + \Omega_e T_{F_i})\right]$$

Iterate until $\left|\theta_N - \theta_{N-1}\right| < .0003$ radians

$$a_j = \sin^{-1}\left[\frac{\cos\theta_N \sin(\phi_T - \phi_L + \Omega_e T_{F_N}}{\sin\theta_N}\right]$$

$$v_L = -(x_o{}^2 + y_o{}^2)^{1/2} \; \Omega_e \cos a_j$$

$$v_d = (x_o{}^2 + y_o{}^2)^{1/2} \; \Omega_e \sin a_j$$

$$v_r = -v_d + KST0 + KST1(\theta_N) + KST2(\theta_N)^2$$

$$T_p = K2ST0 + K2ST1(v_r) + K2ST2(v_r)^2$$

$$AZ_L = a_j + \frac{\frac{v_L}{v_r}}{1 - \frac{T_p}{3T_{F_N}}}$$

## Initialization

**Launch Azimuth Estimation** (Cont'd)

$K_o = 220.37$

$K_1 = 20.621$

$K_2 = 0.$

KSTO = 11137.6

KST1 = 199.4406

KST2 = -.848166

K2STO = 141.675

K2ST1 = .0062625

K2ST2 = 0.

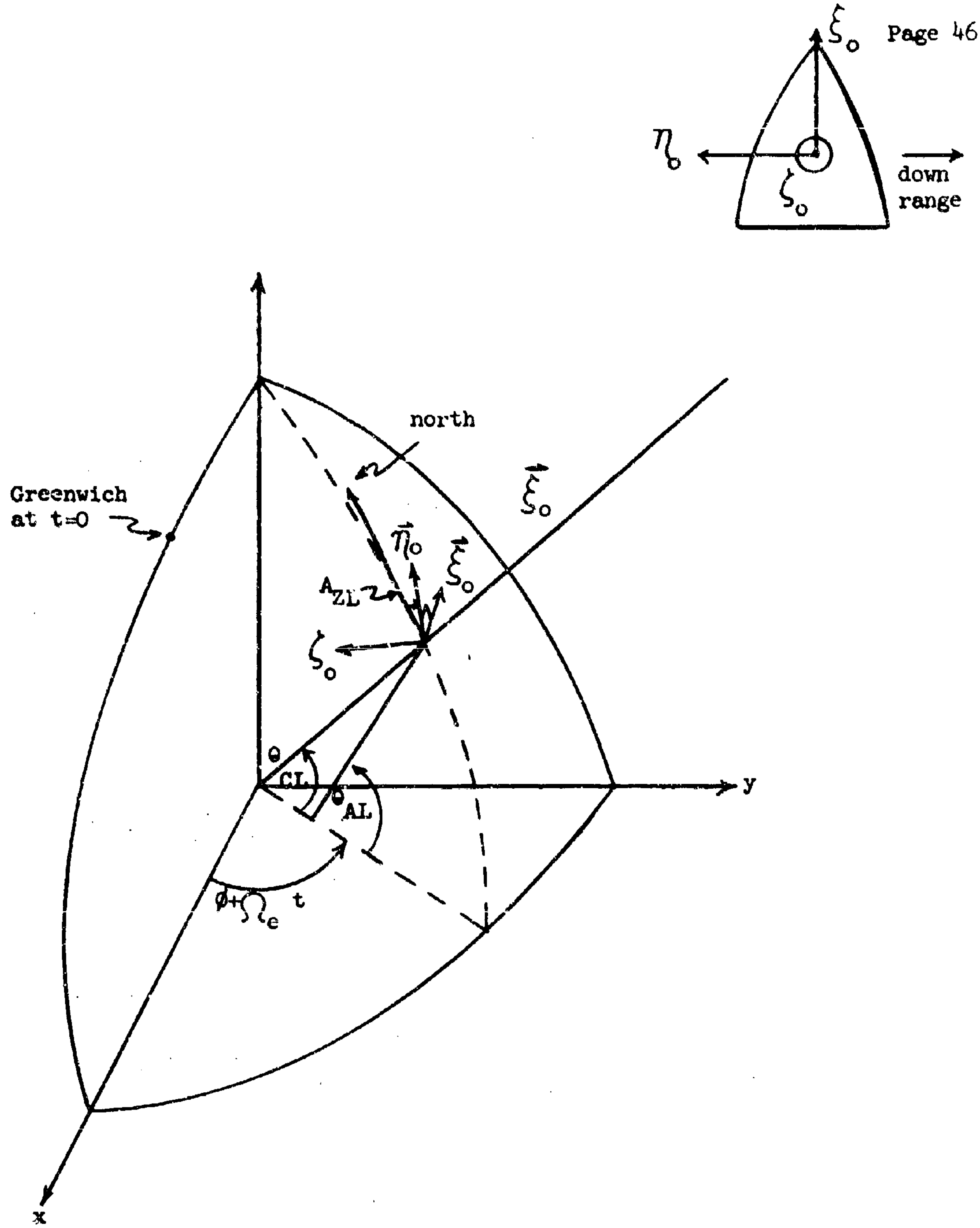
**Launch Attitude**

To achieve launch vertical the $\vec{\xi}, \vec{\eta}, \vec{\zeta}$ system is oriented such that $\vec{\xi}$ points along the x direction, $\vec{\eta}$ along the z direction, and $\vec{\zeta}$ in the negative y direction. The $\vec{\xi}, \vec{\eta}, \vec{\zeta}$ system is rotated through longitude of launch, astronomic latitude of launch, and the supplement of launch azimuth. (Fig. 15)

$$\vec{\xi}'' = \vec{\xi} \cos(-\phi_L - \Omega_e t) + \vec{\zeta} \sin(-\phi_L - \Omega_e t)$$

$$\vec{\eta}'' = \vec{\eta}$$

$$\vec{\zeta}'' = -\vec{\xi} \sin(-\phi_L - \Omega_e t) + \vec{\zeta} \cos(-\phi_L - \Omega_e t)$$




INITIAL ATTITUDE VECTORS

FIGURE 15

Initialization

Launch Attitude (Cont'd)

then

$$\vec{\xi}' = \vec{\xi}'' \cos(-\theta_{AL}) - \vec{\eta}'' \sin(-\theta_{AL})$$

$$\vec{\eta}' = \vec{\xi}'' \sin(-\theta_{AL}) + \vec{\eta}'' \cos(-\theta_{AL})$$

$$\vec{\zeta}' = \vec{\zeta}''$$

finally

$$\vec{\xi}_0 = \vec{\xi}'$$

$$\vec{\eta}_0 = \vec{\eta}' \cos(AZ_L - 180) - \vec{\zeta}' \sin(AZ_L - 180)$$

$$\vec{\zeta}_0 = \vec{\eta}' \sin(AZ_L - 180) + \vec{\zeta}' \cos(AZ_L - 180)$$

Launch coordinate rotational quantity, $\mu_L$ is estimated, if not specified, as

$$\mu_L = 180 - AZ_L$$

Position and Velocity Components at Launch

Sea level radius of launch (Fig. 16) is

$$r_{SL_L} = \bar{A} \left[ \frac{1 + (1-e)^4 \tan^2\theta_{DL}}{1 + (1-e)^2 \tan^2\theta_{DL}} \right]^{1/2}$$

Initialization

Position and Velocity Components at Launch (Cont'd)

Components of $\vec{r}_o$ and $\vec{V}_o$ are

$$x_o = \cos(\phi_L + \Omega_e t) \left[ r_{SL_L} \cos\theta_{CL} + (h_L + g_{SL}) \cos\theta_{DL} \right]$$

$$y_o = \sin(\phi_L + \Omega_e t) \left[ r_{SL_L} \cos\theta_{CL} + (h_L + g_{SL}) \cos\theta_{DL} \right]$$

$$z_o = r_{SL_L} \sin\theta_{CL} + (h_L + g_{SL}) \sin\theta_{DL}$$

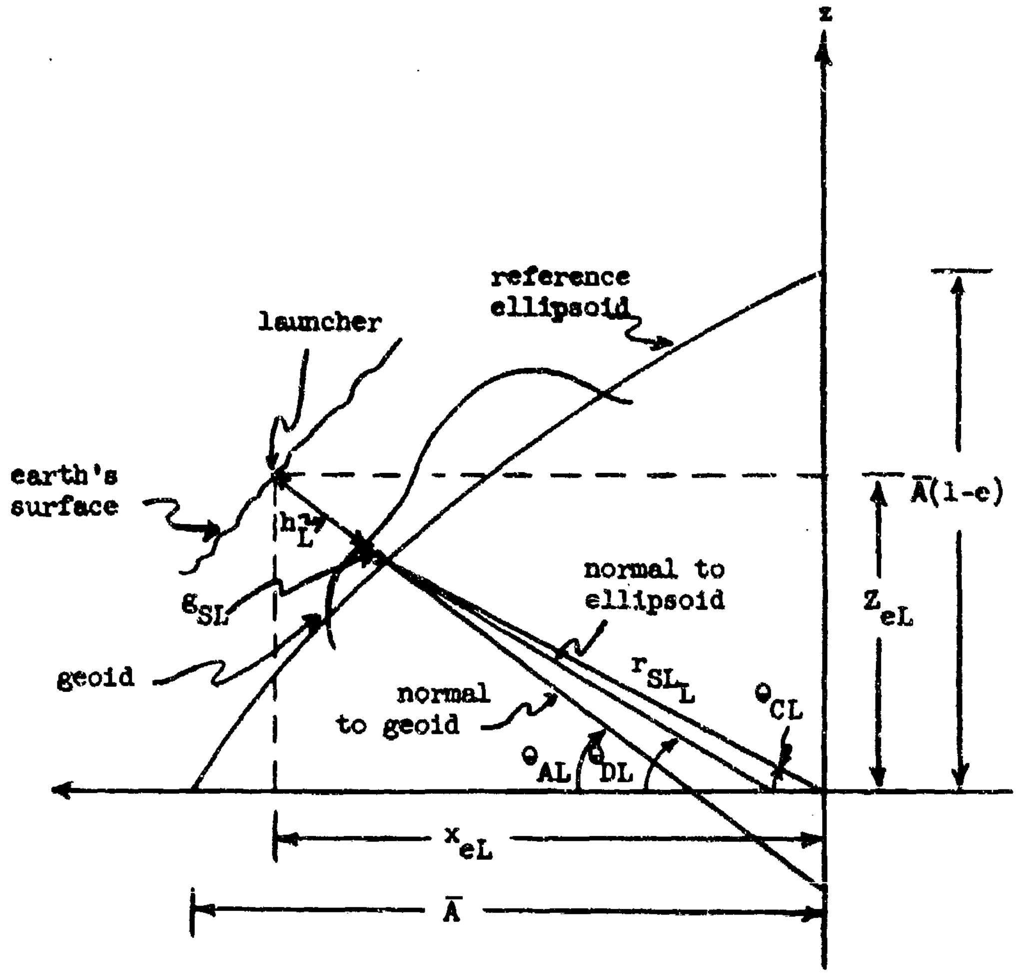
$$\dot{x}_o = -y_o \Omega_e$$

$$\dot{y}_o = x_o \Omega_e$$

$$\dot{z}_o = 0$$

The simulation may be started at an arbitrary point in space with $\vec{r}_o, \vec{V}_o$, $\vec{\xi}_o$, and $\vec{\eta}_o$ specified.

If the position and velocity components are given in a radar coordinate system, the following conversions are made:




LAUNCH COORDINATES ON REFERENCE ELLIPSOID

FIGURE 16

## Initialization

### Radar to Cartesian Coordinate Conversion

#### Ellipsoid - Spherical Coordinates and Doppler Rates to Fixed Rectangular

$$u = R \sin E \sin A$$

$$v = R \sin E \cos A$$

$$w = R \cos E$$

$$R_o = \left[ R^2 + K_4 - uK_1 - vK_2 - wK_3 \right]^{1/2}$$

$$R_1 = \left[ R^2 + K_8 - uK_5 - vK_6 - wK_7 \right]^{1/2}$$

$$R_2 = \left[ R^2 + K_{12} - uK_9 - vK_{10} - wK_{11} \right]^{1/2}$$

$$p = \frac{K_{16} + 2(uK_{13} + vK_{14} + wK_{15})}{R_o + R_1}$$

$$q = \frac{K_{20} + 2(uK_{17} + vK_{18} + wK_{19})}{R_o + R_2}$$

$$\dot{R}_o = \dot{R}$$

$$R_o \, \dot{p} = R_p \, \dot{p} + p(\dot{R}_o - \dot{p})$$

$$R_o \, \dot{q} = R_o \, \dot{q} + q(\dot{R}_o - \dot{q})$$

$$D = K_{23}(u - K_{24}) + K_{22}(v - K_{25}) + K_{21}(w - K_{26})$$

Initialization

Radar to Cartesian Coordinate Conversion (Cont'd)

$$\dot{u} = \frac{R_o\dot{q}}{D}\left[K_{15}(v-K_{25})-K_{14}(w-K_{26})\right] + \frac{R_o\dot{p}}{D}\left[K_{18}(w-K_{26})-K_{19}(v-K_{25})\right] + \frac{R_o\dot{R}_o}{D}\cdot K_{23}$$

$$\dot{v} = \frac{R_o\dot{q}}{D}\left[K_{13}(w-K_{26})-K_{15}(u-K_{24})\right] + \frac{R_o\dot{p}}{D}\left[K_{19}(u-K_{24})-K_{17}(w-K_{26})\right] + \frac{R_o\dot{R}_o}{D}\cdot K_{22}$$

$$\dot{w} = \frac{R_o\dot{q}}{D}\left[K_{14}(u-K_{24})-K_{13}(v-K_{25})\right] + \frac{R_o\dot{p}}{D}\left[K_{17}(v-K_{25})-K_{18}(u-K_{24})\right] + \frac{R_o\dot{R}_o}{D}\cdot K_{21}$$

Doppler radar coefficients (see Fig. 13).

$K_1 = 2(u_o)$

$K_2 = 2(v_o)$

$K_3 = 2(w_o)$

$K_4 = (u_o)^2 + (v_o)^2 = d_o^{\,2}$

$K_5 = 2(u_1)$

$K_6 = 2(v_1)$

$K_7 = 2(w_1)$

$K_8 = (u_1)^2 + (v_1)^2 + (w_1)^2 = d_1^{\,2}$

Initialization

Radar to Cartesian Coordinate Conversion (Cont'd)

$K_9 = 2(u_2)$

$K_{10} = 2(v_2)$

$K_{11} = 2(w_2)$

$K_{12} = (u_2)^2 + (v_2)^2 + (w_2)^2 = d_2{}^2$

$K_{13} = u_1 - u_o$

$K_{14} = v_1 - v_o$

$K_{15} = w_1 - w_o$

$K_{16} = d_o{}^2 - d_1{}^2$

$K_{17} = u_2 - u_o$

$K_{18} = v_2 - v_o$

$K_{19} = w_2 - w_o$

$K_{20} = d_o{}^2 - d_2{}^2$

$K_{21} = K_{13}K_{18} - K_{17}K_{14}$

$K_{22} = K_{15}K_{17} - K_{19}K_{13}$

$K_{23} = K_{14}K_{19} - K_{18}K_{15}$

$K_{24} = u_o$

$K_{25} = v_o$

$K_{26} = w_o$

Initialization

Radar to Cartesian Coordinate Conversion (Cont'd)

Spherical coordinates and rates to ellipsoid-fixed rectangular coordinates

$$u = R \sin E \sin A$$

$$v = R \sin E \cos A$$

$$w = R \cos E$$

$$\dot{u} = \dot{R} \sin E \sin A + R\dot{E} \cos E \sin A + R\dot{A} \sin E \cos A$$

$$\dot{v} = \dot{R} \sin E \cos A + R\dot{E} \cos E \cos A - R\dot{A} \sin E \sin A$$

$$\dot{w} = \dot{R} \cos E - R\dot{E} \sin E$$

Site Fixed Cartesian Coordinate and Rate Transformation to Inertial System

$$\begin{bmatrix} \xi \\ \eta \\ \zeta \end{bmatrix} = \begin{bmatrix} \cos\mu_R & -\sin\mu_R & 0 \\ \sin\mu_R & \cos\mu_R & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} u \\ v \\ w \end{bmatrix}$$

$$\begin{bmatrix} \dot{\xi} \\ \dot{\eta} \\ \dot{\zeta} \end{bmatrix} = \begin{bmatrix} \cos\mu_R & -\sin\mu_R & 0 \\ \sin\mu_R & \cos\mu_R & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{u} \\ \dot{v} \\ \dot{w} \end{bmatrix}$$

$$\alpha = \xi \qquad \dot{\alpha} = \dot{\xi}$$

$$\beta = \eta - \delta_{w_R} \cdot \zeta \qquad \dot{\beta} = \dot{\eta} - \delta_{w_R} \cdot \dot{\zeta}$$

$$\gamma = \delta_{w_R} \cdot \eta + \zeta \qquad \dot{\gamma} = \delta_{w_R} \cdot \dot{\eta} + \dot{\zeta}$$

Initialization

Radar to Cartesian Coordinate Conversion (Cont'd)

$$\begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix} = \begin{bmatrix} \sin\theta_{DR} & 0 & \cos\theta_{DR} \\ 0 & 1 & 0 \\ -\cos_{DR} & 0 & \sin\theta_{DR} \end{bmatrix} \begin{bmatrix} \alpha \\ \beta \\ \gamma \end{bmatrix} + \begin{bmatrix} x_{er} \\ 0 \\ z_{er} \end{bmatrix}$$

$$\begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{z}_e \end{bmatrix} = \begin{bmatrix} \cos\theta_{DR} & 0 & \sin\theta_{DR} \\ 0 & 1 & 0 \\ -\sin\theta_{DR} & 0 & \cos\theta_{DR} \end{bmatrix} \begin{bmatrix} \dot{\alpha} \\ \beta \\ \dot{\gamma} \end{bmatrix}$$

$$\vec{r} = \begin{bmatrix} \cos(\Omega_e t + \delta\phi_R & -\sin(\Omega_e t + \delta\phi_R) & 0 \\ \sin(\Omega_e t + \delta\phi_R & \cos(\Omega_e t + \delta\phi_R) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_e \\ y_e \\ z_e \end{bmatrix}$$

$$\vec{V} = \begin{bmatrix} \cos(\Omega_e t + \delta\phi_R) & -\sin(\Omega_e t + \delta\phi_R) & 0 \\ \sin(\Omega_e t + \delta\phi_R) & \cos(\Omega_e t + \delta\phi_R) & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \dot{x}_e \\ \dot{y}_e \\ \dot{z}_e \end{bmatrix} + \Omega_e \begin{bmatrix} -y \\ x \\ 0 \end{bmatrix}$$

Attitude from Angles of Attack

The following conversions are made to initialize the attitude of the missile when $\alpha_p$ and $\alpha_y$ are known:

Initialization

Attitude from Angles of Attack (Cont'd)

$$\vec{V}_a = \vec{V} + \Omega_e \begin{bmatrix} y \\ -x \\ 0 \end{bmatrix}$$

$$\vec{\xi}_o' = \frac{\vec{V}_a}{|\vec{V}_a|}$$

$$\cos\beta = \frac{1}{|\vec{r}|} \left[ x \cdot \xi_{x_o}' + y \cdot \xi_{y_o}' + z \cdot \xi_{z_o}' \right]$$

$$\sin\beta = \left[ 1 - \cos^2\beta \right]^{1/2}$$

$$\vec{\eta}_o' = \left[ \frac{\vec{r}}{|\vec{r}|} - \cos\beta \cdot \vec{\xi}_o' \right] \frac{1}{\sin\beta}$$

$$\vec{\zeta}_o' = \vec{\zeta}_o' \times \vec{\eta}_o'$$

$$\vec{\xi}_o = \cos\alpha_y \left[ \cos\alpha_p \cdot \vec{\xi}_o' - \sin\alpha_p \cdot \vec{\eta}_o' \right] + \sin\alpha_y \cdot \vec{\zeta}_o'$$

$$\vec{\eta}_o = \sin\alpha_p \cdot \vec{\xi}_o' + \cos\alpha_p \vec{\eta}_o'$$

$$\vec{\zeta}_o = \vec{\xi}_o \times \vec{\eta}_o$$

## Initialization

<u>Tie Down</u>

Derivatives of equations of motion to maintain vertical attitude during launch hold down are computed from

$$\omega_r = -(\xi_z) \cdot \Omega_e$$

$$\omega_p = -(\zeta_z) \cdot \Omega_e$$

$$\omega_y = -(\eta_z) \cdot \Omega_e$$

$$\dot{\vec{V}} = -(\Omega_e)^2 \cdot \begin{bmatrix} x \\ y \\ 0 \end{bmatrix}$$

# APPENDICES

# APPENDIX A

## Symbol Definitions

| | |
|---|---|
| $\bar{A}$ | Semi-major axis of reference ellipsoid. |
| $A_{Z_L}$ | Azimuth of launch, measured clockwise from north. |
| $A_{Z_w}$ | Wind azimuth, measured clockwise from north. |
| $\bar{B}$ | Semi-minor axis of reference ellipsoid. |
| $C$ | Speed of sound. |
| $C_D$ | Tabular drag coefficient. |
| $e$ | Ellipticity of reference ellipsoid = $\frac{\bar{A}-\bar{B}}{\bar{A}}$ . |
| $\epsilon$ | Eccentricity = $\frac{\bar{C}}{\bar{A}}$, where $\bar{C}$ is the distance from the center of the ellipsoid to a focus along the major axis. |
| $G$ | Gravitational Acceleration. |
| $\phi$ | Longitude. |
| $\theta_A$ | Astronomic latitude, angle between the equitorial plane and the local direction of gravity. |
| $\theta_C$ | Geocentric latitude, angle between the equatorial plane and $\vec{r}$. |
| $\theta_D$ | Geodetic latitude, angle between the equatorial plane and the normal to the reference ellipsoid. |
| $K_F$ | Multiplier for tabular thrust $F_o$. |
| $K_p$ | Wind perturbation multiplier. |
| $K_{\dot{M}}$ | Multiplier for tabular mass flow rate $\dot{M}_o$. |
| $K_{\omega_{pt}}$ | Multiplier for tabular pitch rate. |
| $K_{\omega_{rt}}$ | Multiplier for tabular roll rate. |
| $K_{\omega_{yt}}$ | Multiplier for tabular yaw rate. |

APPENDIX A (Cont'd)

| | |
|---|---|
| $M$ | Mass of missile. |
| $M_I$ | Reference initial mass for ablation computation. |
| $N_m$ | Mach number. |
| $P$ | Atmospheric pressure. |
| $Q$ | Dynamic pressure. |
| $\rho$ | Atmospheric density. |
| $\vec{r}$ | Missile position vector whose components are x,y,z. |
| $r_o$ | Average radius of geoid (spherical approximation). |
| $S$ | Missile cross-sectional area. |
| $T$ | Atmospheric temperature. |
| $\phi$ | Longitude. |
| $V_w$ | Tabular wind velocity. |
| $\dot{x},\dot{y},\dot{z}$ | Components of missile's velocity vector $\vec{V}$. |
| $\Omega_e$ | Reference ellipsoid's rotation rate. |
| $\omega_p,\omega_y,\omega_r$ | Angular rotational rate about respectively pitch, yaw and roll axes. |
| $\omega_{p_c},\omega_{y_c},\omega_{r_c}$ | Respectively pitch, yaw and roll rates from the guidance program. |
| $\omega_{p_d},\omega_{y_d},\omega_{r_d}$ | Respectively pitch, yaw and roll drifts. |

# APPENDIX B

## Iteration

Two general schemes of iteration are used. The first, called (for lack of a better name) "paired function", requires that each parameter to be determined be paired with a function to be constrained. The second form of iteration, termed "matrix", does not require pairing of the functions and parameters but does not allow minimizing or maximizing of functions.

In either type of iteration, control of the portions of a trajectory over which iteration is to take place is provided. The control system allows "nesting" of "paired function" sets within other "paired function" or "matrix" sets. This capability allows combination of the strong points of the two general schemes according to the needs of the individual problem.

### Paired Functions

After an initial trajectory is integrated, a perturbation is added to the parameter in question and another trajectory is integrated. Inverse La Grangian interpolation or extrapolation is then used to estimate a new value for the parameter. (The desired value of the function is used as argument.) The trajectory is again integrated and the results enable the use of a higher order polynomial in the interpolation or extrapolation procedure. The process is used repeatedly until convergence. The polynomial order may be limited by input, or in any case at order 5.

If minimization or maximization is called for, a second perturbation is added to the parameter and a third trajectory is produced. La Grangian interpolation is then used in a "step-and-bisect" iterative procedure which determines minimum or maximum for the polynomial fitting the available points. If no minimum or maximum is available, another perturbation is added to the parameter. In either case another trajectory is produced and the new point is used to enable a higher order polynomial to be used in the "step-and-bisect" solution. Again the La Grangian polynomial order may be limited by input, or at order 5.

**Paired Functions** (cont'd)

The "step-and-bisect" process is said to converge when the improvement in the function is less than one half the (input) accuracy requirement. The function maximization (minimization) is complete when the difference between output of the "step-and-bisect" procedure and the previous value of the function is less than the (input) accuracy requirement.

**MATRIX Iteration**

The Newton-Raphson method for iterative linear differential correction may be stated in its matrix form:

$$(1)\quad \begin{bmatrix} -\Delta Y \end{bmatrix} = \begin{bmatrix} \frac{\partial Y_1}{\partial X_1} & \frac{\partial Y_1}{\partial X_2} & \cdots & \cdot \\ \frac{\partial Y_2}{\partial X_1} & & & \vdots \\ \vdots & & \cdots & \frac{\partial Y_m}{\partial X_n} \end{bmatrix} \begin{bmatrix} \Delta X \end{bmatrix}$$

Where $[\Delta Y]$ consists of the differences from desired conditions, and $[\Delta X]$ is the suggested changes to the peremeters in question.

The partial derivative elements $\frac{\partial Y_i}{\partial X_j}$ may be replaced by their finite difference equivalents, $\frac{\Delta Y_{i,j}}{\Delta X_j}$. In the N-STAGE program, in order to enable the use of the Generalized Secant Method, the resultant equation is re-written as two equations in the equivalent form:

$$(2)\quad \begin{bmatrix} -\Delta Y \end{bmatrix} = \begin{bmatrix} \Delta Y_{1,1} & \Delta Y_{1,2} & \cdots & \cdot \\ \Delta Y_{2,1} & & & \vdots \\ \vdots & & \cdots & \Delta Y_{m,n} \end{bmatrix} \begin{bmatrix} P \end{bmatrix}$$

$$(3)\quad \begin{bmatrix} \Delta X \\ \\ \\ \end{bmatrix} = \begin{bmatrix} \Delta X_1 & 0 & \cdots & \cdot \\ 0 & \Delta X_2 & & \vdots \\ \vdots & & & \\ \cdot & \cdots & & \Delta X_n \end{bmatrix} \begin{bmatrix} P \\ \\ \\ \end{bmatrix}$$

Where $\Delta Y_{i,j}$ indicates the difference in $Y_i$ produced by a change $\Delta X_j$ in $X_j$. The elements $\Delta Y_{i,j}$ are obtained by repeatedly integrating the trajectory with each of the parameters in question perturbed in its turn. The elements $\Delta X_j$ of the diagonal matrix (3), are the perturbation values. The elements of $[\Delta Y]$ and $[\Delta Y_{i,j}]$ are divided by normalizing elements $N_i$ (which are input quantities) in order to scale, or establish dimensional homogeneity.

The first equation is solved for $[P]$ in a "least squares"* sense, which allows the general case, $m \neq n$. The multiplication implied in the second equation is then performed, yielding the set $[\Delta X]$, the elements of which are added to the appropriate parameters.

After a solution of the above sort is accomplished, and the trajectory representing the suggested parameters integrated and found to be still unconverged, the Generalized Secant Method may be utilized to improve the convergence without integrating the number of trajectories (n), necessary to recompute the matrices for equations (2) and (3). In this method a new column, consisting of the negative of the normalized (scaled) deviations from conditions at the previous nominal trajectory integration, is appended to each matrix as the first column, and is added to each of the remaining columns, producing matrices with one more column than the previous matrices.

$$(4)\quad \begin{bmatrix} \\ -\Delta Y \\ \\ \end{bmatrix} = \begin{bmatrix} -\Delta Y_{1,o} & \Delta Y_{1,1} - \Delta Y_{1,o} & \cdots & \cdot \\ & & & \vdots \\ -\Delta Y_{2,o} & & & \vdots \\ \vdots & \cdots & & \Delta Y_{m,n} - \Delta Y_{m,o} \end{bmatrix} \begin{bmatrix} \\ P \\ \\ \end{bmatrix}$$

*See SWLS4F, General Linear Equation Solver by D. D. Morrison, Sept. 24, 1959 (CDRC computer subroutine).

$$(5)\quad \begin{bmatrix} \Delta X \end{bmatrix} = \begin{bmatrix} -\Delta x_{1,o} & \Delta x_{1,1} - \Delta x_{1,o} & \cdots & \cdot \\ -\Delta x_{2,o} & -\Delta x_{2,o} & & \vdots \\ \vdots & \cdots & & \Delta x_n - \Delta x_{n,o} \end{bmatrix} \begin{bmatrix} P \end{bmatrix}$$

The process may be used repeatedly, each time defining the previous trajectory to be the previous nominal trajectory.
The last column may be deleted after any number of additional columns have been accumulated.

A satisfactory rationale for deletion or non-deletion of the last column is not presently available, but experience indicates that the last column should be kept if the number of functions is approximately equal to the number of parameters (m=n), and deleted when the number of functions is markedly greater than the number of parameters (m>n). Tests with m and n initially equal to 3 have shown a 20% increase in efficiency when the matrices are allowed to expand indefinitely.

In particular, when inaccuracies due to system non-linearities (round-off error accumulations, integration truncation error, staging condition annomalies, etc.) are on the order of the iteration convergence accuracies required, it has been found necessary to allow the matrices to expand in order to gain convergence.

An additional advantage of the Generalized Secant Method is that perturbation delta magnitudes assume a less dominant role in convergence efficiency. In many cases convergence is worsened by a Newton-Raphson solution because of erroneous partial derivative information due to improper perturbation sizes. In such cases it is not uncommon to find that the Generalized Secant Method can proceed to convergence.

The Generalized Secant Method is used repeatedly, until the gain in one step is less than the input quantity SCGN. (The gain is measured as the ratio of the summations of the squares of the elements of the normalized error vector $\begin{bmatrix}\Delta Y\end{bmatrix}$, for two successive solution evaluation passes.) Thus, if any solution pass (whether the solution comes from Newton-Raphson or Generalized Secant Method) is not materially better than the previous solution or nominal pass, the program recomputes the matrices by perturbation methods. Note that if SCGN is a very large number, the Generalized Secant Method is not attempted.

Eigenvalues associated with the solution of equation (2) or (4) are printed following each evaluation. These Eigenvalues are of use only if it is understood that they are for the matrix transpose multiplied by the matrix, and it should be remembered that the elements are scaled. The ratio of the largest Eigenvalue to the smallest (known as the condition number) is sometimes a clue to the degree of linear dependence in the matrix. A possibly confusing issue is that the overall perturbation in the column associated with a particular Eigenvalue also affects the condition number. Zero Eigenvalues should not be found for a Newton-Raphson solution, but are often found for a Generalized Secant Method solution. In fact, usually only (n) Eigenvalues are large enough to have any effect, even though the matrix has been allowed to expand considerably larger than the original (n) columns.

In any case, the Eigenvalues are associated with input parameters in reverse order to their printing. That is, the last Eigenvalue is associated with the first iteration variable.

A matrix iteration is determined to have converged on either of two criteria.

1. The error in each of the functions is less than the corresponding accuracy requirement.
2. The change in the error (for each of the functions which fails the first criterion) is less than one half the corresponding accuracy requirement. The change is measured from the previous solution or the nominal trajectory.

# APPENDIX C

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